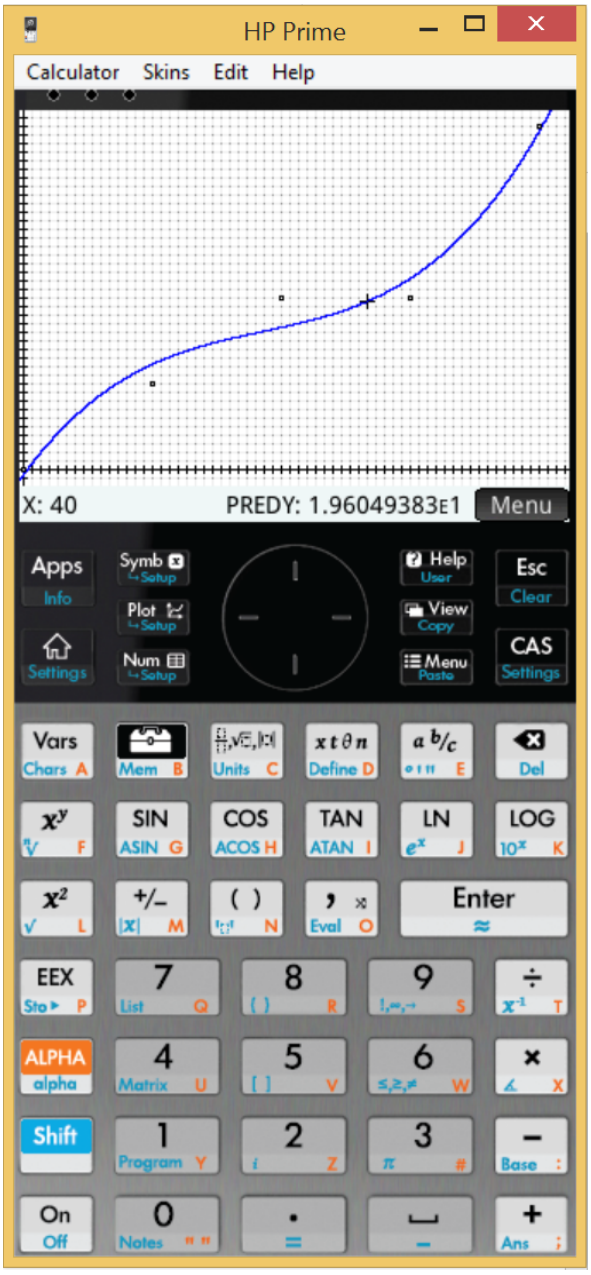
Mathematical Methods: Units 1&2
Prime activities

Using technology to support mathematics learning

Ian Sheppard

Chris Longhurst



Mathematical Methods: Units 1&2 – HP Prime activities
Using technology to support mathematics learning

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Table of Contents

Introduction:

This book comprises a series of activities which are designed to facilitate learning about both the technology (HP Prime) and the mathematics. It is written as a student workbook.

Unlike a textbook, the activities cover neither the whole course, nor are they restricted to purely course material. Activities beyond the course content can assist students with solving problems within the course while also increasing the ability to explore broader mathematical questions, including further mathematics study. In contrast to many electronic device manuals this book is about mathematics with detailed instructions on how the technology can be used.

The activities vary in the time needed to complete them. Some are primarily concerned with how to perform a particular technique, and some use the Prime's output as the starting point. In others, the Prime is only a small part of the activity.

The activities are arranged into chapters matching the Australian Curriculum topics. Within each topic the activities reflect a possible sequence of learning related to that topic. Many activities can be used as a precursor to formal teaching of the concept thus encouraging a sense-making approach.

Each activity has an aim, linking to curriculum documents, the activity itself and usually a section of *Learning notes*. Fully worked solutions are provided at the end of the text. The learning notes are intended to help with the understanding of concepts, provide more detail or help with instructions for Prime use, provide additional explanations or point to interesting further explorations. As the course progresses more assumptions are made about the skills students have developed and so the instructions become briefer. Where more detailed instructions are required on Prime use, it will often be in the *Learning notes* rather than in the text of the investigation.

The Computer Algebra System (CAS) is very powerful but can also be frustrating. When doing algebraic manipulation with pen and paper, mathematicians often use the current line of working to determine the next step. Using CAS, however, requires the articulation of steps in words and these words are then the commands for CAS to perform the next step. *Solve*, *simplify*, *factor* and *expand* are examples of these words. Generally, the result is useful, but sometimes there may not be a suitable command. In these circumstances it may be necessary to work with part of an expression, or even return to pen and paper.

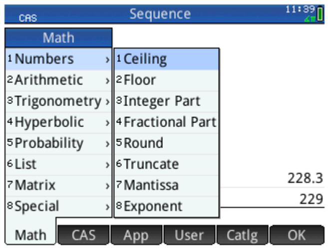
Knowing when Prime use is quicker or more efficient becomes easier the more experience students have. Working through the activities will help you learn this.

CAS enables us to do is to focus more on what we want to do rather than how do we do it. For example, in a modelling situation we may come across awkward functions that students do not yet have the tools to deal with by traditional methods. Often, however, CAS will provide a means of calculating an answer so the result can be evaluated in the context of the situation.

A lot of detail has been provided in the Prime instructions. However, it is impractical to cover all possible arrangements and settings. These activities were written for the Prime.

In the instructions:

- *Press* refers to a key on the Prime;
- *Tap* is an option displayed on the touch screen;
- A sequence of menu options is shown in the form **Math > Numbers > Ceiling**



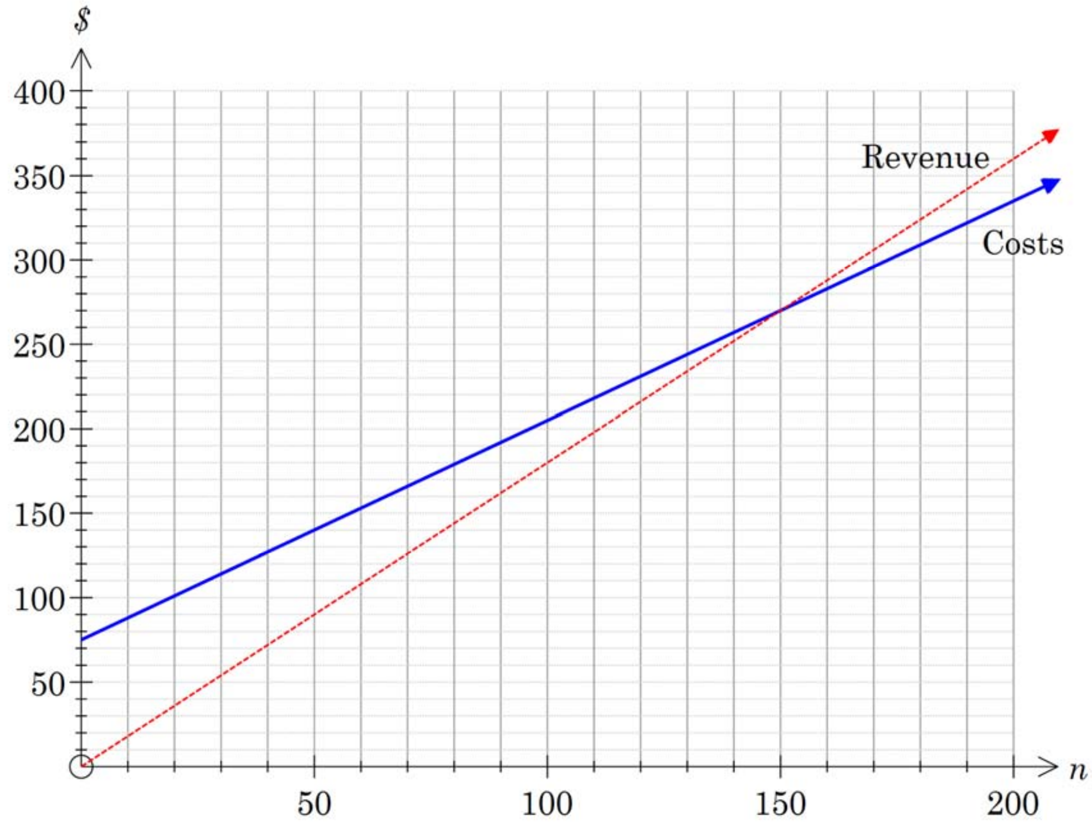
It is advisable to:

- check the settings are appropriate, e.g. Number format, angle measure;
- become familiar with the soft keyboard and where to find commands;

These materials have been adapted from
Mathematical Methods Units 1&2: ClassPad activities
by Sheppard and Pateman 2004.

Chapter 1 Functions and Graphs

Investigation	Key concepts
Features of graphs	Recognise and describe key features of graphs
How big is the package?	Modelling with a cubic function
Circles	Equations of circles
Phone costs	Function notation



Activity 1 Features of graphs

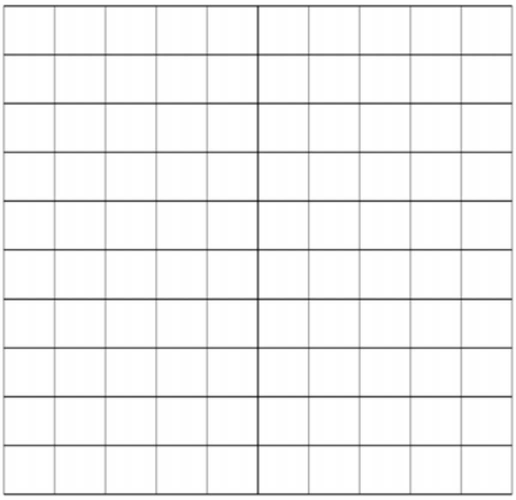
Aim: Identify and determine key features of functions from their graphs.

Graphs have particular features. In this investigation you will identify some of these features.

For each function:

- Graph the function.
- Label the indicated features for each graph.
- Record the coordinates of the point or describe the feature.
- Which features can you connect with the equation?

1. $y = 7 - 2x$

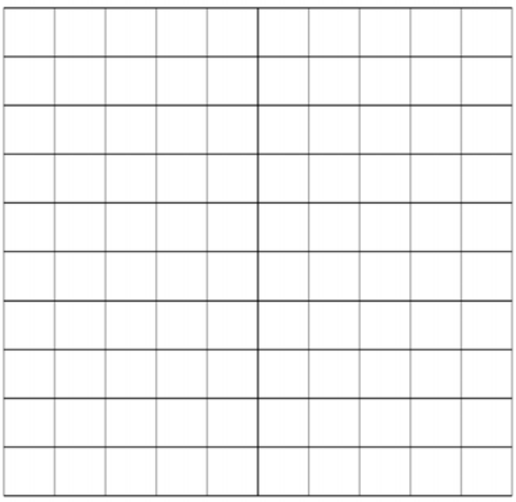


a) y-intercept

b) x-intercept(s)

c) Gradient

2. $y = -2(x+1)(x-3)$

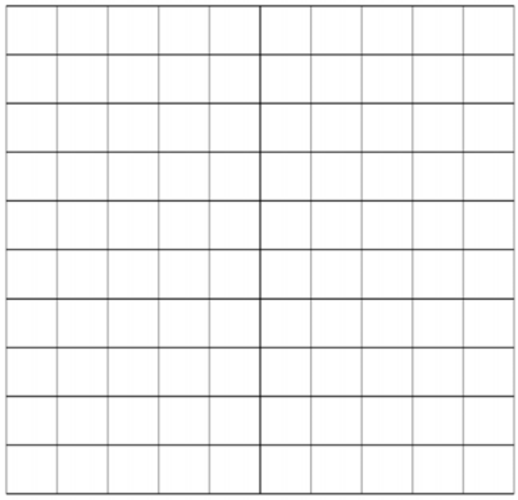


a) y-intercept

b) x-intercept(s)

c) coordinates of turning point

3. $y = -2(x+1)^2(x-3)$

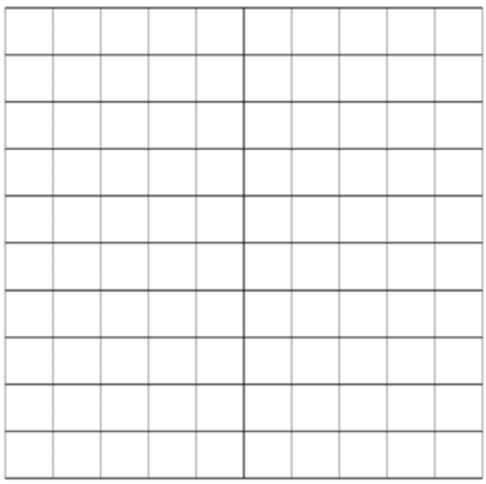


a) y-intercept

b) x-intercept(s)

c) coordinates of turning point(s)

4. $y = (x-2)(x+1)(x+3)$

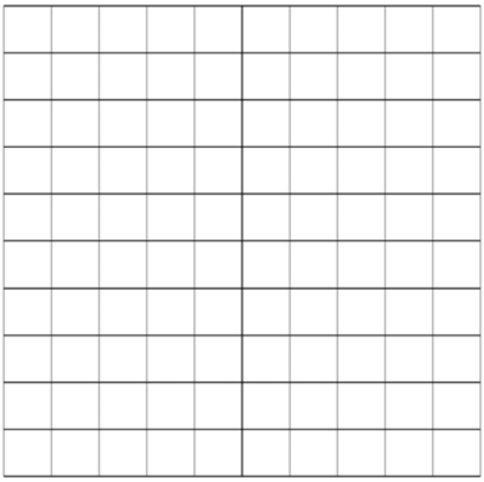


a) y-intercept

b) x-intercept(s)

c) coordinates of turning point(s)

5. $y = 1.3^x$

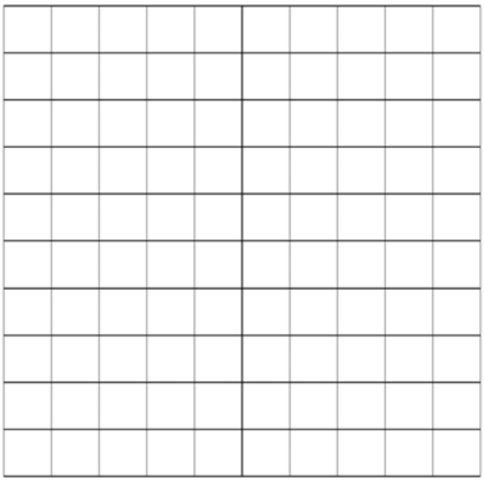


a) y-intercept

b) x-intercept(s)

c) Equation of horizontal asymptote

6. $y = \dfrac{2}{x+1}$



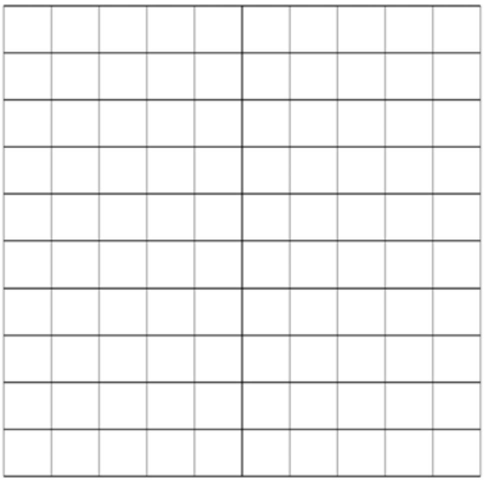
a) y-intercept

b) x-intercept(s)

c) Equation of horizontal asymptote

d) Equation of vertical asymptote

7. $y = 0.2(x + 2.5)^2$



a) *y*-intercept

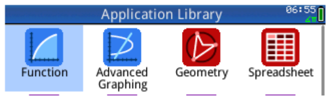
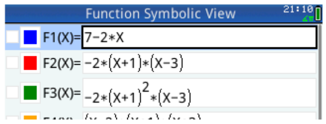
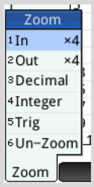
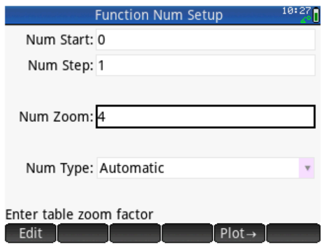
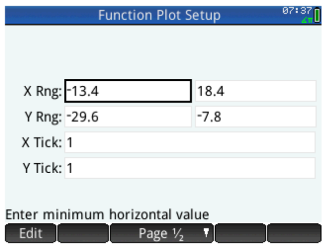
b) *x*-intercept(s)

c) coordinates and nature of turning point(s).

Learning Notes

The functions in this activity go beyond those specified in this topic. In this topic you will be expected to identify the relevant features without the aid of technology too.

To draw a graph with Prime

Draw a graph • Press Apps (Info) • Select Function	
Enter the function • Press 7, ×, 2, x t θ n for $y = 7 - 2x$ • Tap OK or press Enter • Press Plot (Setup) to draw the checked graphs	
Display a table of values • Press Num (Setup)	
Change *x*-values displayed • Press Shift Num (Setup) to set up start number and step • Press Num (Setup) Note you can make further adjustments using Zoom 	
Adjust the window size • pinch and pull on screen **Set the view window** • Press Shift Plot (Setup) Set the desired boundaries of the view window. When you wish to record the graph and the scales have been given, then set the boundaries to match your graph.	

To record the graph on paper:

- Consider the values needed to display the graph.
- Position and draw in the axes. The origin does not need to be in the centre of the grid or bottom left corner.
- Decide on your scale and label the axes. It is desirable to have the graph as large as possible that fits on the grid.
- Plot key points to ensure reasonable accuracy.
- Sketch the graph.

Calculating values

View coordinates along the graph

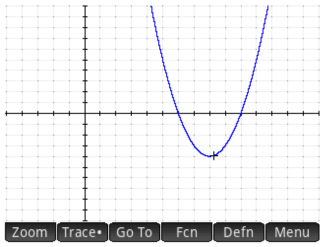
- Tap TRACE• (you may need to toggle Menu)
  This enables you to read approximate values, good for transcribing the graph

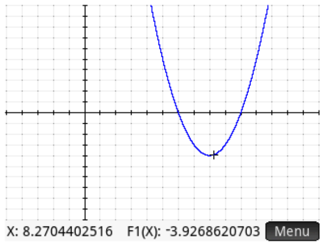
Locate *x*-intercept(s)

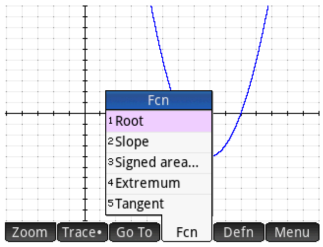
- Look at the table of values
  Or
- Move cursor and read values on screen
  Or
- Tap Fcn and select **1Root**

Locate local maximum and minimum turning points

- Tap Fcn and select **4Extremum**
  (Note Prime will only locate the closest turning point. Reposition the curser and search again to find others.)





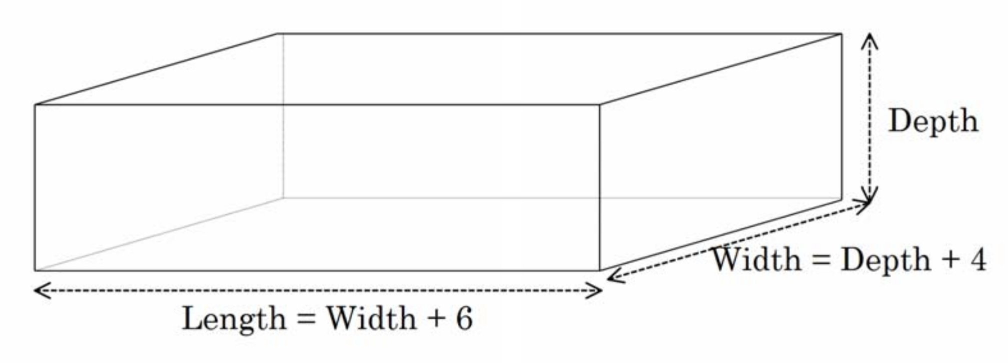


Prime will not locate asymptotes for you. In this course they are often integers and so you can easily read them from the graph. Vertical asymptotes have equations of the form x = a number and horizontal asymptotes have equations y = a number. Changing the zoom or size of the window may also be helpful.

Activity 2 How big is the package?

Aim: Construct graphs of a cubic function and solve cubic equations.

Matt makes and sells model dolls. When Matt gets an order he makes the model and builds a box to send the model to the customer.



The length of the box is 6 cm longer than the width which is 4 cm longer than the depth of the box.

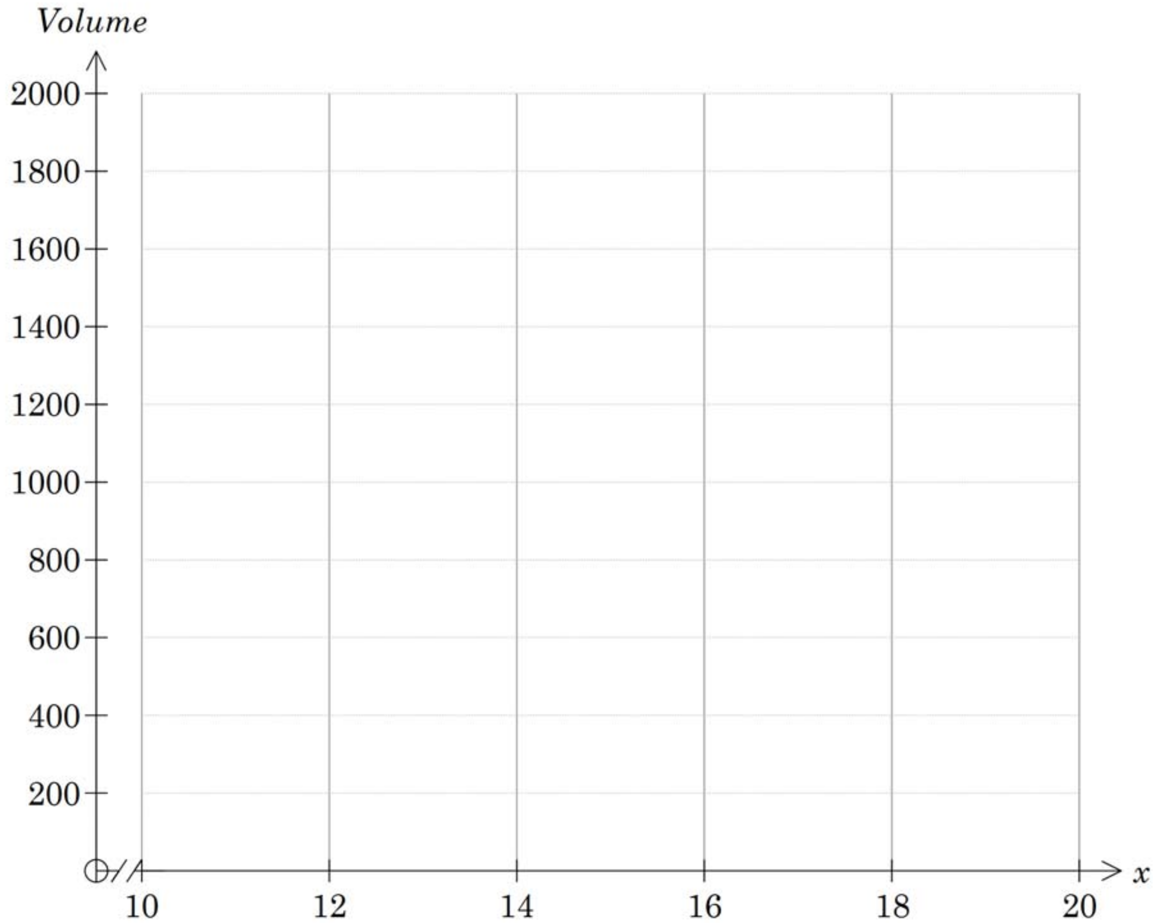
1. Complete the table to calculate the volume of various boxes

Box	Length (cm)	Width (cm)	Depth (cm)	Volume (cm^3)
A			5	
B		16		
C	25			
D				144

2. Explain why the volume, V, of the box of length x cm is given by the equation $V = x(x-6)(x-10)$.

3. Why is $x > 10$?

4. Draw the graph on Prime. Plot sufficient points to make a reasonably accurate plot. A suitable table of values will help.
(See Learning notes for detailed instructions)



5. Determine the:

a) volume of a box of length 17.43 cm

b) volume of a box of width 32.7 cm

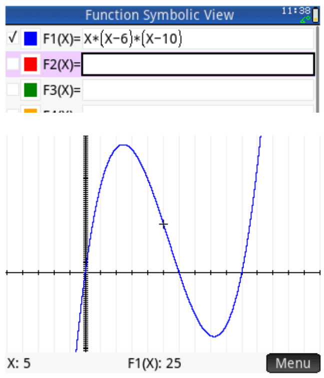
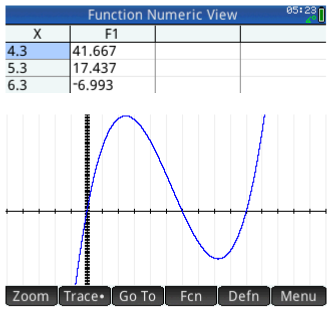
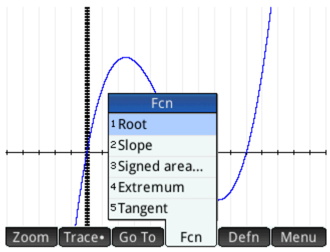
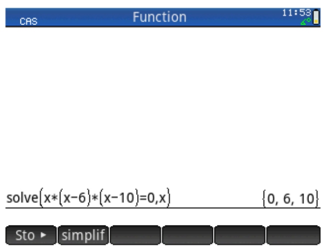
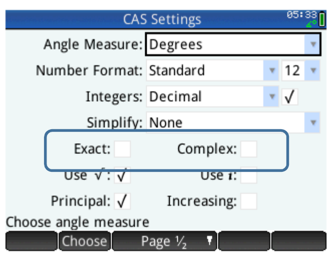
c) length of box with volume 2.5 L

d) dimensions of a box with volume in excess of 2750 cm^3

Learning Notes

Q1 d) A trial and error approach is sufficient.

In this investigation you are working with a cubic equation derived by calculating the volume of a box.

Q4 Draw the function • Press Symb Apps and select Function • Enter the function as F1(X) (You can press shift clear to clear all functions. If no make sure the on you wish to graph is checked) • Press Enter • Press Plot to draw the graph	
Calculate *y*-value(s) • Press Num and type the desired *x*-value and press Enter Or • On graph screen tap Menu (if required) then tap Go To and type the *x*-value and press Enter	
Calculate roots or *x*-intercepts • Tap Menu then Fcn • Tap on **1root** to get other roots move the cursor near the other root and repeat	
Calculate *x*-values • Press CAS then • Tap CAS, select **3Solve** then **1Solve** • enter equation followed by x and Enter equation can be like F1(x)=2500,x	
Settings • Exact: unticked	

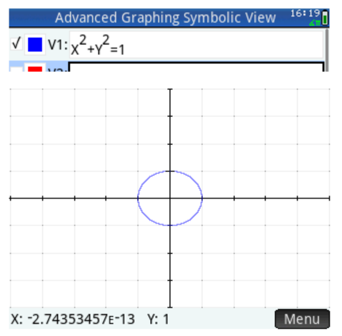
Activity 3 Circles

Aim: Investigate the equations of circles.

Graph a circle in Advanced Graphing app

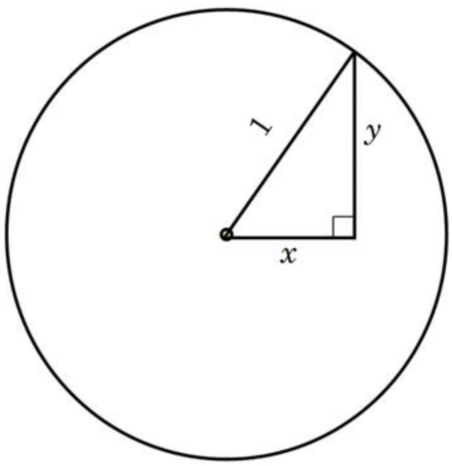
(See Learning notes for more detailed instructions)

- Open the Advanced Graphing app
- Insert equation $x^2 + y^2 = 1$
- Press Enter Plot



This suggests the equation of a unit circle centred at the origin has Cartesian equation $x^2 + y^2 = 1$. Is this what you expected? Why should this be the case?

1. Use the diagram to show that the equation makes sense.



2. Experiment with different values for the radius of the circle (maintaining the centre at the origin) and note the resulting equations. Generalise this for a circle with radius *r* units.

What about circles centred elsewhere?

3. Draw the following circles in the Advanced Graphing app and complete the table

Equation	Centre	Radius
$(x-1)^2+y^2=1$		
$(x-2)^2+(y-1)^2=1$		
$(x+1)^2+(y+3)^2=4$		
$(x-A)^2+(y-B)^2=R^2$		

4. Complete the square for the x terms in the equation below. The equation represents the circle with radius 1 unit, centred at (3,0).

$$x^2-6x+y^2=-8$$
$$(x-\square)^2-\square+y^2=-8$$
$$(x-\square)^2+y^2=\square$$

5.

a) Predict the completed square form of a circle with radius 4 units centred at (–2, 3).

b) Expand and simplify, then check your answer using the Advanced Graphing app.

6. Determine the centre and radius for the circle with equation $x^2-5x+y^2+8y-13.75=0$

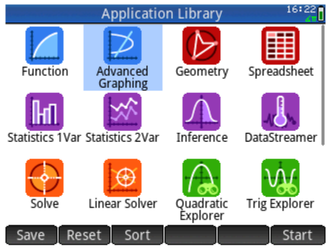
Use the Advanced Graphing app to check your result

Learning notes

Using The Advanced Graphing App

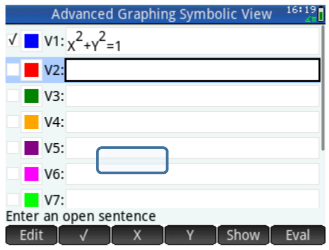
Open Advanced Graphing

- Press Apps (Info)
- Select Advanced Graphing
- Press Enter



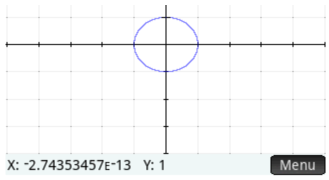
Enter an equation

- Select the equation e.g. V1
- Use the X and Y buttons across the bottom of the screen to enter x and y in your equation.



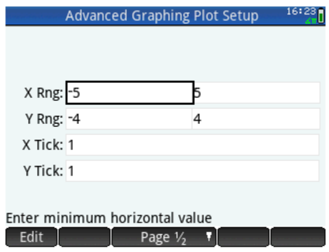
To Plot a curve and change screen size

- Press Plot (Setup)



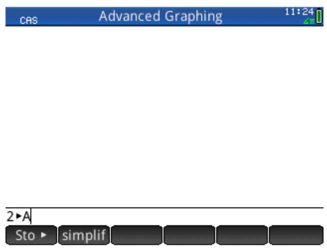
To change the graph window

- Press Shift Plot (Setup)
- Enter the desired range and domain
- Press Plot (Setup)



Set values for A and B (Q3)

- Press CAS (Settings) to
- Enter the value, tap Sto ▸ and enter the variable (Use capital letter (Shift ALPHA (alpha)) for a variable)
- Press Enter
- Press Plot (Setup)



Activity 4 Phone costs

Aim: Use and interpret function notation.

Suzie's pre-paid account with *FourMobile* has $250 value. The table below shows how Suzie is charged for her calls.

Local rates per minute (?)	
Call rate per minute or part thereof	$ 0.89
Flagfall rate per call	$ 0.39

1. Study Suzie's call records listed in the following table.

Date	Time	Phone Number	Duration	Call minutes
1/3/12	4:17		6:54	7
1/3/12	4:24		18:25	19
1/3/12	5:11		0:05	1
1/3/12	5:11		0:42	1
2/3/12	5:12		12:15	13
2/3/12	6:12		2:00	2
4/3/12	3:59		17:01	18
4/3/12	7:05		1:12	2
4/3/12	7:29		21:34	22

a) How many calls has Suzie made?

b) What is the total number of call minutes Suzie will be charged for?

c) What is the cost of Suzie's calls (including flag fall and rate per minute costs)?

d) How much of the $250 credit does Suzie have left?

The credit remaining on this \$250 plan is a function of the number of calls, n and the number of call minutes, m.

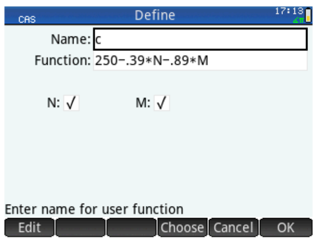
$$c(n,m) = 250 - 0.39n - 0.89m\,.$$

For example after 20 calls and 100 call minutes the remaining credit is $c(20,100) = 250 - 0.39 \times 20 - 0.89 \times 100 = \$153.20$.

2. Complete the table.

	Number of calls	Call minutes	Credit remaining (\$)
$c(10,250)$			
$c(50,150)$			
	72	175	
$c(32,\quad)$		220	
$c(\quad, 200)$			\$56.40

3. What is the maximum number of calls that could have been made if there were 250 call minutes?

Define the function in Prime • Press CAS Settings to open CAS • Press Shift $xt\theta n$ Define then ALPHA alpha Units C and press Enter ≈ to call the function c • Use the keyboard to enter 250–0.39N–0.89M for the expression. (Use Shift ALPHA alpha ( ) N to enter N) and tap Enter ≈ (Use capital letters for variables in Prime functions)	
Evaluate function • Press and enter the values given E.g. enter c(10,20) to find the credit after 10 calls and 20 call minutes	

4. Use your Prime function to answer the following questions.

   a) What is the credit remaining after 72 calls and 240 call minutes?

   b) What is the credit remaining after 16 calls and 250 call minutes?

   c) Suzie checks her balance and notices it is \$45.26 and that she has made 64 calls. How many call minutes has Suzie made?

5. Record the Prime output for the following inputs:

a) $c(10, m)$

b) $c(10, \text{mins})$

c) $c(x, y)$

d) $c(10, 2m)$

e) $c(x, 2y)$

6. Suzie's remaining credit will also take into account charges for standard national SMS texts (t) and excess data charges (d).

Standard national SMS	$ 0.29
Excess data usage fee (per MB)	$ 2.00

a) Write the function rule for

$c(n, m, t, d) =$

b) Modify or redefine your Prime function and complete the table.

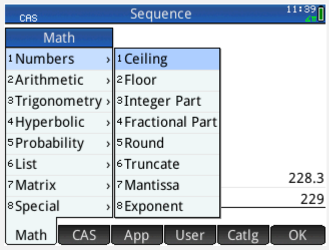
	Number of calls	Call minutes	SMS	Excess Data (Mb)	Remaining Credit ($)
$c(10,150,75,0)$					
$c(10,90,350,3)$					
	72	175	21	4	
$c(32,100,60,\quad)$					$107.12
	21		73	0	$43.53

EXTENSION

FourMobile would want call minutes calculated automatically. It would be calculated using the integer part of a number function.

On Prime CEILING returns the smallest integer greater than or equal to the input. For example CEILING(228.3) returns 229.

In CAS mode:
press , select **Math > Numbers > Ceiling**



7. Determine the value for each of the following function statements and compare with the table in Q1.

   a) CEILING(6.54)

   b) CEILING (18.25)

   c) CEILING (0.05)

   d) CEILING (0.42)

   e) CEILING (12+15/60)

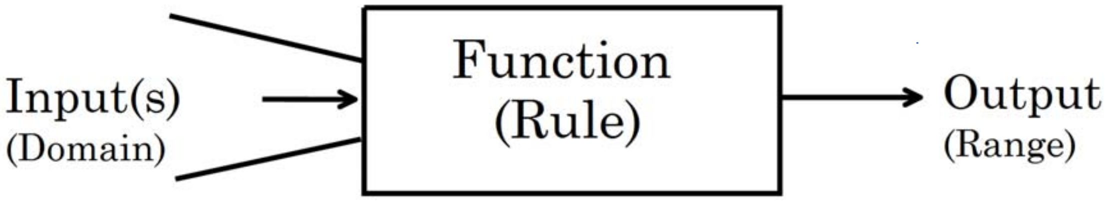
   f) CEILING (2.00)

8. Define a function to calculate call minutes given the duration of a call as a decimal.

Learning Notes

Mathematical functions involve one or more inputs that generate one output or value. For example y-values of a function graph depend upon x.

In three dimensions a z-value is likely to be a function of x and y.

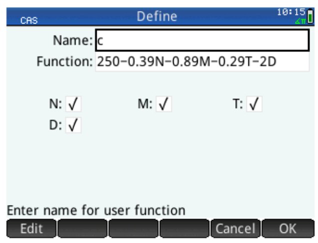
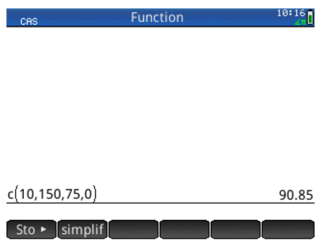


The Credit function in this investigation depends upon two factors: number of calls and call minutes. This assists in providing a realistic context to explore function notation and to appreciate that functions produce a single output.

Most of the functions you will study in this course are single variable functions. This topic includes linear, quadratic and cubic functions.

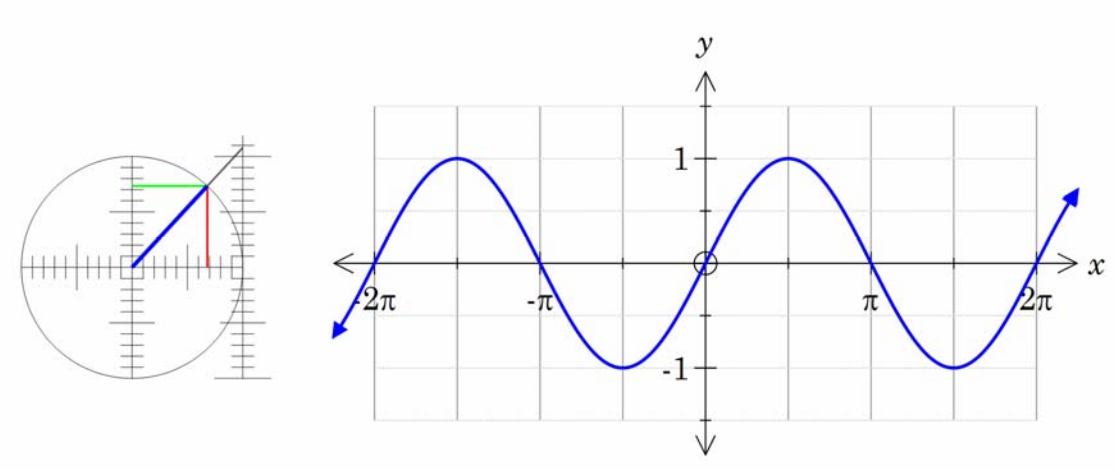
Functions in Prime:
Avoid single capital letters for function names as these are already set up as variables.

Q6

Define the function with 4 variables • Press CAS to open CAS • Press Shift xtθn and press Enter to call the function the function c • Use the keyboard to enter 250–0.39N–0.89M–0.29T–2D for the expression. (Use Shift ALPHA to enter variables as capital letters) and tap Enter)	
Evaluate function • In CAS window enter the function name • enter the values given E.g. enter c(10,150,75,0) to find the credit after 10 calls, 150 call minutes, 75 SMS's and 0 Mb of extra data.	

Chapter 2 Trigonometric Functions

Activity	Key concepts
Trigonometric graph transformations	Examine amplitude, period and phase changes in trigonometric graphs
Modelling with trigonometric functions	Model practical situations using trigonometric functions
Window dressing	Solve problems involving non-right triangles



Activity 5 Trigonometric graph transformations

Aim: Modify equations to investigate transformations of the basic trigonometric functions.

This activity uses the Trig Explorer App.

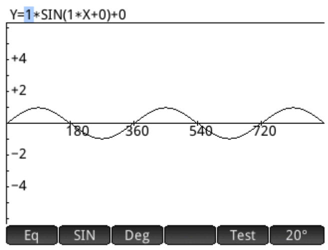
Setup

- Open Trig Explorer App
- Ensure Prime is in degree mode

  Tap Deg / Rad to switch between degrees and radians

Adjust parameters

- The function $y = a\sin(b(x+h)) + v$ appears at the top of window in the form Y=1*(1*X+0)+0
- Press left and right to change highlighted parameter
- Press up or down arrow to change parameter value



Controls

- Eq / Graph to switch modes
- SIN / COS to switch between sin and cosine graphs
- You might explore the other options too.

With our initial values for the parameters, $a = 1$, $b = 1$, $h = 0$ and $v = 0$, we have displayed the graph of $y = \sin x$.

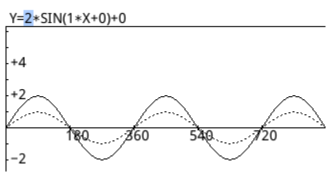
1. Describe the main features of the graph of $y = \sin x$ i.e. x- and y- intercepts, period and amplitude.

For Q's 2 – 12 use terms such as translation, dilation and reflection when describing changes to the graphs.

2. Describe the effect of a on the graph of $y = a\sin x$.

Modify the parameter *a*

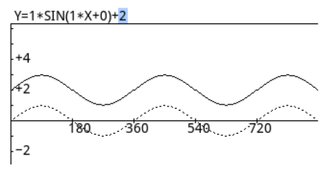
- Highlight the parameter a in the equation and up and down arrow to increase its value.



3. Describe the effect of v on the basic graph of $y = \sin x + v$.

Modify the parameter *v*

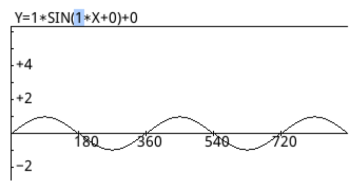
- Set *a* to 1
- Highlight the parameter v.
  Adjust its value using the arrows.



4. Describe the effect of b on the basic graph of $y = \sin bx$.

Modify the parameter *b*

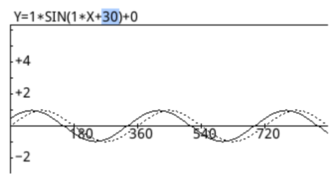
- Set *v* to 0
- Highlight the parameter b
  Adjust its value using the controller arrows.



5. Describe the effect of h on the basic graph of $y = \sin(x + h)$. Return the value of h to 0 when finished.

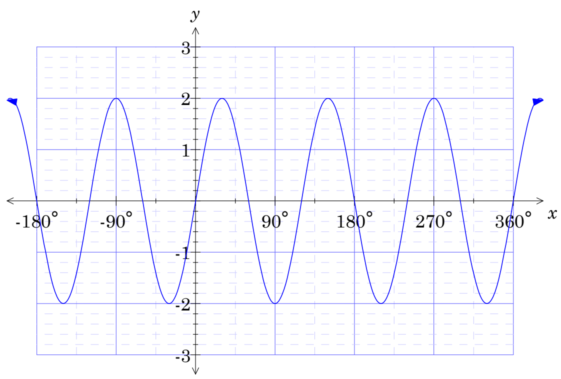
Modify the parameter *h*

- Set *b* to 1
- Highlight the parameter h
  Adjust its value using the controller arrows

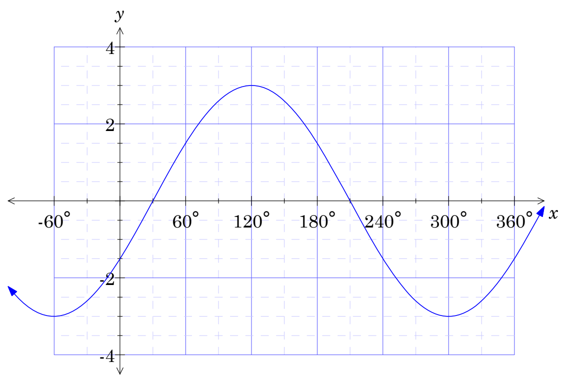


6. Determine equations for the following sine graphs.

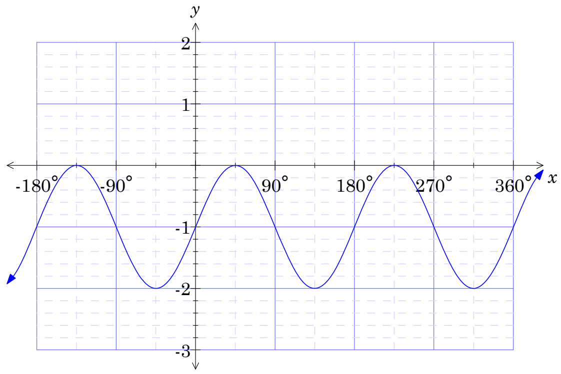
a)



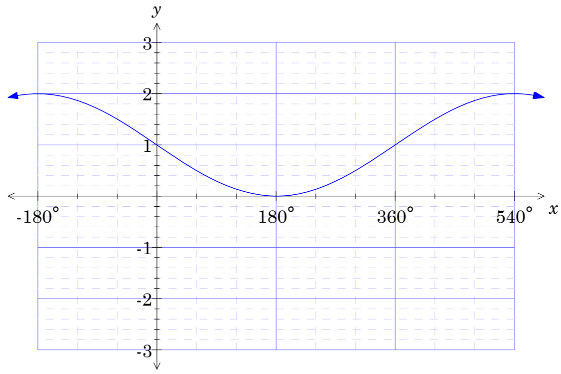
b)



c)



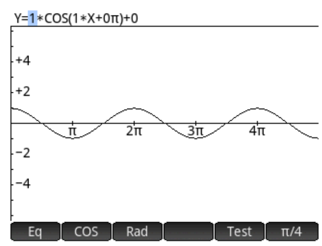
d)

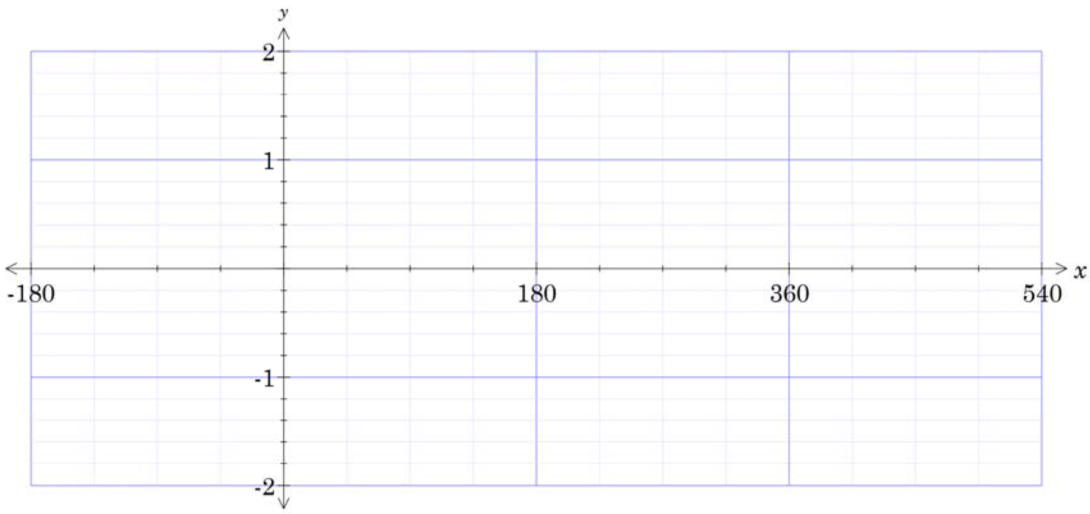


7. Sketch the graph of $y = \cos x$ on the axes below showing key features.

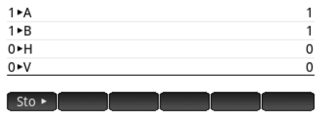
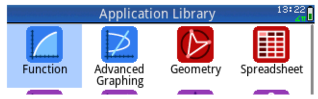
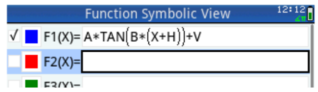
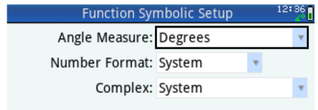
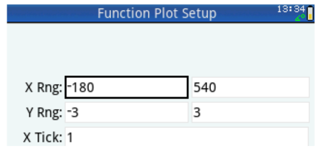
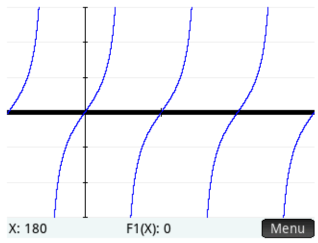
Investigate graph of the Cosine function

- Tap SIN / COS to switch the equation to $a \cdot \cos(b \cdot (x + h)) + v$

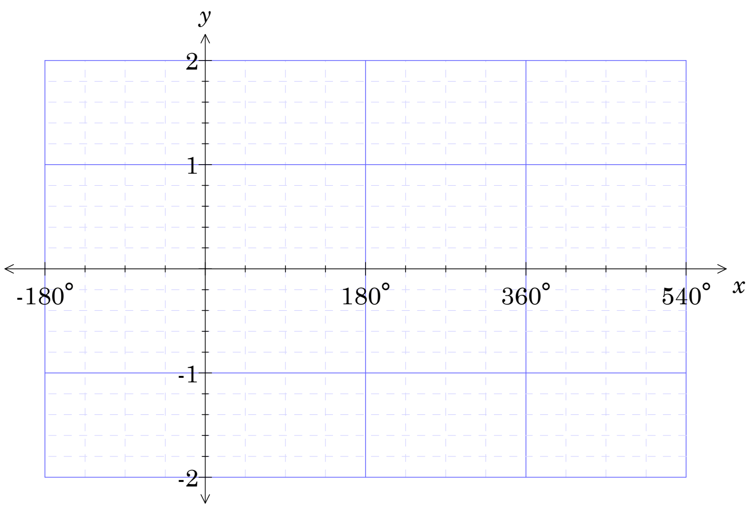




8. **Investigate tan graph manually**
Use the Sto ▸ in Settings to change the values of a, b, h and v.
Note that a Step of 1 should be used for all except h. How do the transformations compare to those of the sine function?

Store parameters A, B, H and V • Press Settings • input 1 Sto ▸ A Enter • 1 Sto ▸ B Enter • 0 Sto ▸ H Enter • 0 Sto ▸ V Enter	
Open Function App • Press Apps Info and Tap Function	
Enter the equation • A*Tan(B*(X+H)+V into F1(X) • Tap OK	
Set angle to degrees • Press Shift Symb Setup • Select Degrees from the pull-down menu	
Set domain and range for graph window • Press Shift Plot Setup and set x and y range $-180 \le x \le 540$ and $-3 \le y \le 3$ as shown	
Plot the graph • Press Plot Setup NOTE: To change to RADIANS mode in Function App Press Shift Symb Setup and change to RADIANS then Press Shift Plot Setup and set x and y range appropriately	

9. Sketch the graph of $y = \tan x$ on the axes below showing key features.

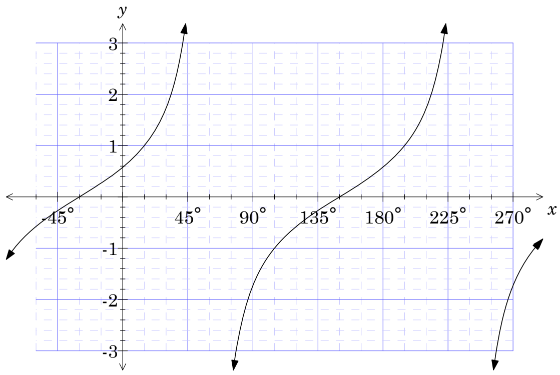


10. Describe the effect on the basic tangent graph of changing each of the parameters A, B, V and H.

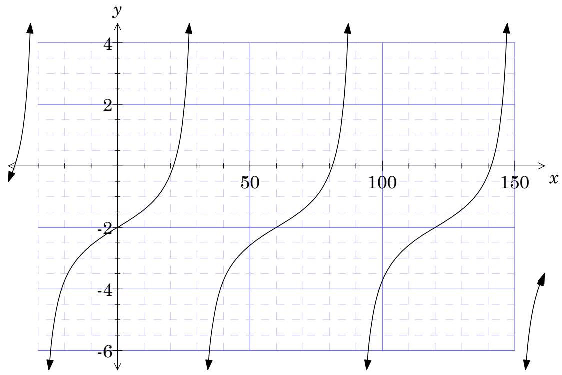
- Note the following suggestions for the Step size:
    - For B and V use 1
    - For H use 15
    - For A use 0.5

11. Determine equations for the following tangent graphs.

a)



b)



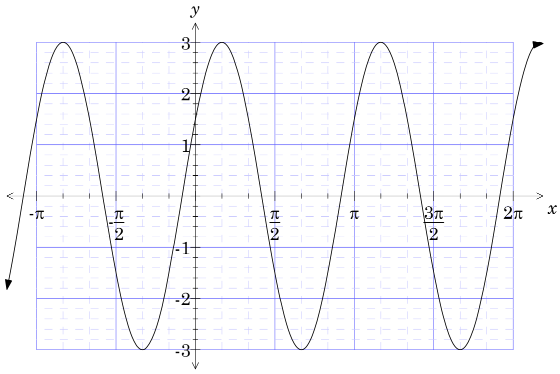
12. Discuss the effects on the sine graph $y = a \cdot \sin(b \cdot (x + h)) + v$ when changing a, b, h and v in radian mode. Try a step size of $\frac{\pi}{6}$ for h.

13. Discuss the effects on the cosine graph $y = a \cdot \cos(b \cdot (x + h)) + v$ when changing a, b, h and v in radian mode.

14. Discuss the effects on the tangent graph $y = a \cdot \tan(b \cdot (x + h)) + v$ when changing a, b, h and v in radian mode.

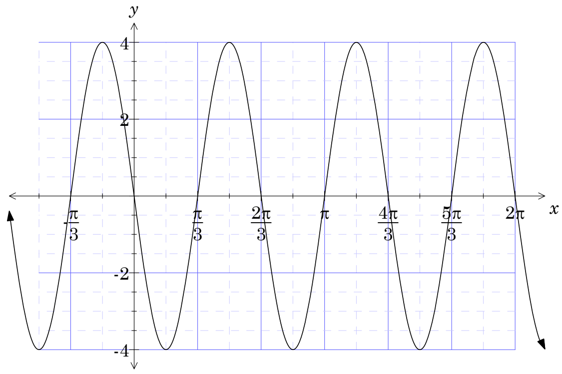
15. Determine equations for each of the following trigonometric graphs.

a)



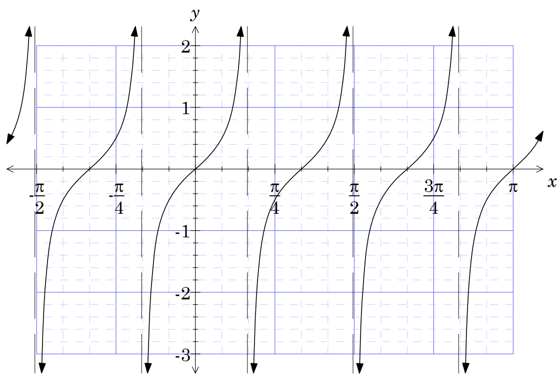
Use cosine

b)



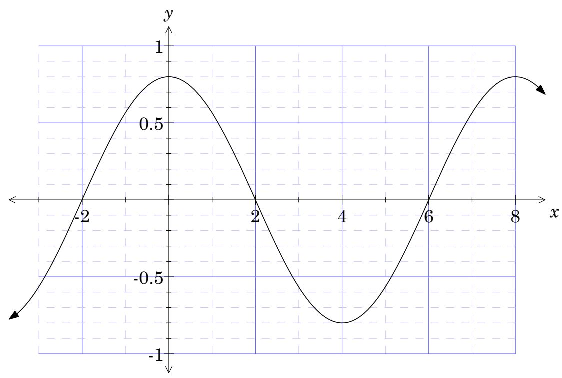
Use sine

c)



Use tangent

d)



Use cosine

Activity 6 Modelling with trigonometric functions

Aim: Identify contexts suitable for modelling by trigonometric functions and use them to solve practical problems.

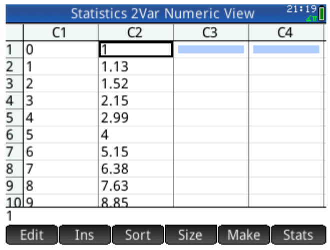
Ferris Wheel

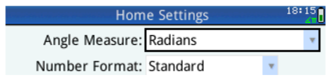
Aaron gets on a Ferris wheel at the Royal Show. His height, h metres, t seconds after the ride starts is given in the table below.

t (s)	0	1	2	3	4	5	6	7	8	9	10
h (m)	1	1.13	1.52	2.15	2.99	4	5.15	6.38	7.63	8.85	10

Model this data to obtain a height function

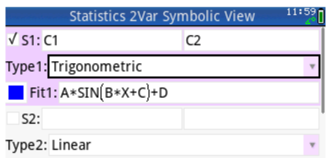
- Press Apps Info
- Select **Statistics 2Var**
- Enter the data
- Ensure angle measure is set to radians
  Note: Shift Settings to change or check





Set graph type

- Press Symb Setup
- Select Trigonometric for graph type.
  (You may need to scroll down in the menu)
- Tap Fit



1.

a) Explain why a trigonometric model would be appropriate for this situation and write down the equation with suitable rounding.

b) Use your model to determine the:

i) radius of the Ferris wheel;

ii) minimum and maximum height of Aaron above the ground; and

iii) time taken for one complete revolution.

2. A cosine function provides a slightly simpler model for Aaron's height over time. Determine the equation of such a model.

3. Bev is also on the Ferris wheel, at a height of 7 metres above the ground when the ride begins. Determine a possible model for Bev's height versus time given she is initially moving toward the ground.

Water in the harbour

4. A particular cargo ship has a draft of 8.8 metres when light (carrying no cargo) and 11.3 metres when fully loaded. The ship is currently light and waiting to enter a port to be loaded for a voyage. The depth of water, d metres, in the port over time can be approximated with a sinusoidal model and the data below represents the depth at various times, t hours since midnight.

t (hours)	0	0.5	1	1.5	2	2.5
d (m)	8.7	8.3	8.1	8	8	8.2

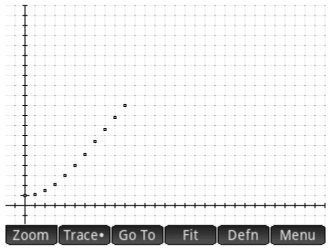
Determine the earliest time the ship can enter the port and the latest time it can safely leave once loaded.

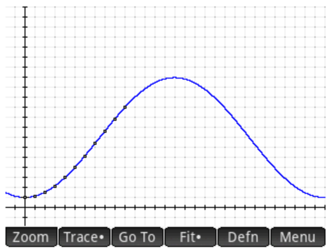
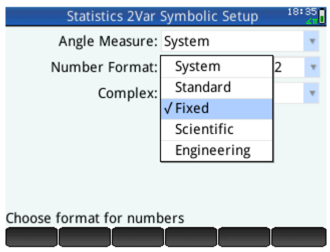
Note: Draft is the distance between the surface of the water and the bottom of a ship's keel.

Learning Notes

By default, the Statistics application is setup to draw scatterplots.

Q4 To solve graphically you could draw another graph such as $y = 8.8$ and find the intersection.

<table>
<tr>
<td>Working with scatterplots in Statistics
• In Statistics 2Var press Plot (Setup)
• Tap FIT• to get line of best fit
• Press Symb (Setup) to look at equation
Tap in the Fit box and tap Show</td>
<td>


</td>
</tr>
<tr>
<td>Change the number of decimal places displayed
• Press Shift Settings and toggle to number format and set the desired number of decimal places.</td>
<td>
</td>
</tr>
</table>

Activity 7 Window dressing

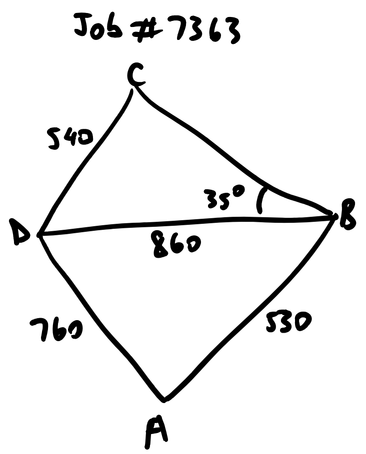
Aim: Solve non-right-angled triangles.

Geometry problems can often be solved by drawing a scale diagram. If using pencil, compass and protractor, we need to draw the diagram sufficiently accurately.

Norman has measured up a window for which glass is to be cut.

This is his rough sketch.

All lengths are in millimetres.



1. Use Triangle Solver App to determine the:

   (Refer to Learning notes for detailed instructions)

   a) size of angle A (or ∠BAD)

   b) size of angle ABD

   c) length of diagonal AC

   d) area of the whole window

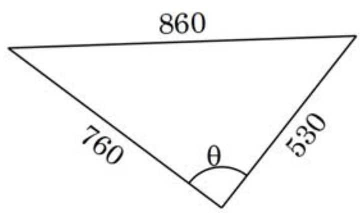
   e) cost of the glass given the glass costs $196.50 per square metre

Your teacher may well want you to use trigonometric formulae in solutions of such problems.

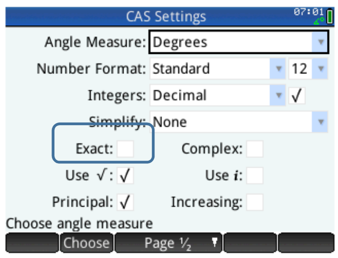
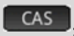
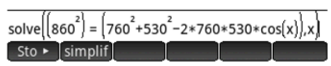
Trigonometric formulae for all triangles	
Area of a triangle	$\text{Area} = \frac{1}{2}ab\sin C$
Sine rule	$\frac{\sin A}{a} = \frac{\sin B}{b}\left(= \frac{\sin C}{c}\right)$
Cosine rule	$c^2 = a^2 + b^2 - 2ab\cos C$

2. With reference to this triangle:

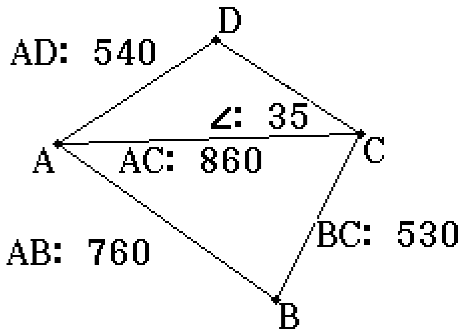
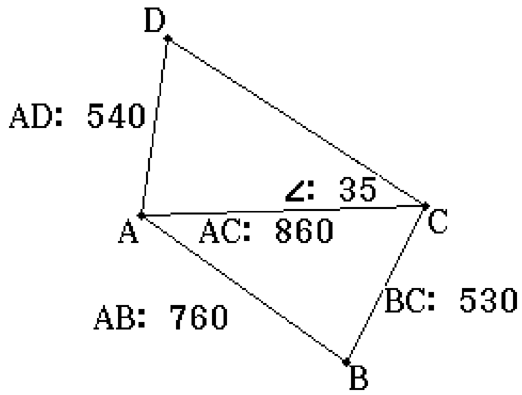
   a) Label the triangle appropriately to use the **cosine rule** to explain why $860^2 = 760^2 + 530^2 - 2\times 760\times 530\cos\theta$



   b) Enter $860^2 = 760^2 + 530^2 - 2\times 760\times 530\cos\theta$ in CAS and solve for θ.

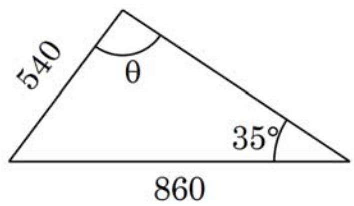
Check settings • Press   • Ensure Angle Measure is Degrees Exact is not checked	
Solve the equation • Open the  screen • Press  • Tap , select **3Solve > 1Solve** and enter the expression shown.	

The glass was cut to specifications by the glazier at the factory and supplied to Norman. Unfortunately it did not fit the frame. The glazier was adamant that he had followed Norman's dimensions exactly. A diagram showing the frame and the supplied glass is shown below.

Frame	Supplied glass
	

3. To understand what went wrong, consider triangle ACD.

   a) Label the triangle appropriately in order to use the **sine rule** to explain why $\frac{\sin\theta}{860} = \frac{\sin 35^\circ}{540}$



   b) Enter this equation in CAS and solve for θ $0^\circ \le \theta \le 180^\circ$. What is the relationship between the two solutions?

   c) Interpret your answer to b) in the context of Norman and the glazier.

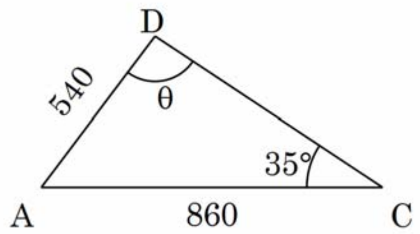
4. The glazier told Norman he could cut the supplied glass to fit the frame.

a) Determine the two possible sizes of angle DAC.

b) Hence describe how the glazier will cut the glass to fit the frame.

Extension

Consider again triangle ACD in the window. If the length AD was not 540, but some other length, would there still be two different sizes of angle ADC?



5.

a) Try a length for AD of 650 mm. What are the two values for angle ADC?

b) What happens when AD is set to 860 mm? What is the significance of this length?

c) There is a length between 490 and 500 mm that is significant.

i) What is this length to 1 decimal place and why is it significant?

ii) Why are lengths smaller than this value not permitted?

Learning notes

A solution is more than an answer. As a minimum a solution requires:

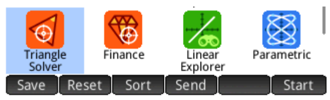
- a labelled diagram;
- an equation with the known values substituted; and
- the answer, appropriately rounded, with units.

For solving equations you have used three methods. It is advisable to use the method that is most efficient for you for each question and this is likely to vary with the problem. The table below gives an indication of advantages and disadvantages of each method.

Method	Advantages	Disadvantages
Using solve in CAS	• You have already written the equation.	• May produce more than one solution
Triangle Solver	• Easy to enter the information and produce all sides and angles	• Can only constrain (set) lengths and angles.

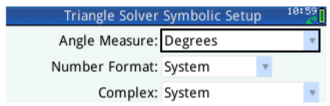
Open Triangle Solver

- Press [Apps Info] and select Triangle Solver



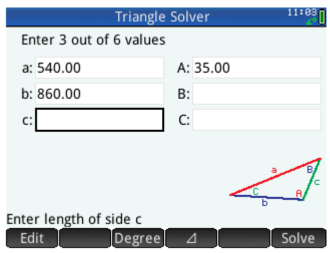
Make sure angle measure is set to degrees

- [Shift] [CAS Settings]
- Select Degrees



Enter triangle measurements

- Enter the triangle information
- Toggle to a blank space and tap [Solve]

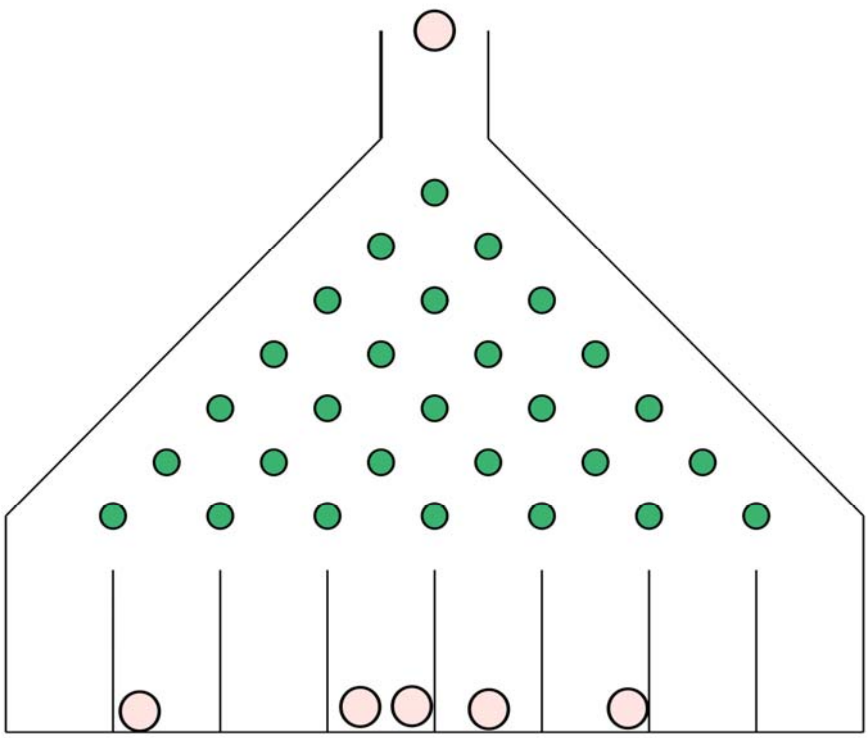


Solve a new triangle

- Select value to clear and press [Del]
- Repeat as required
- Enter new value(s)

Chapter 3 Counting and probability

Investigation	Key concepts
Pascal's triangle	Generate Pascal's triangle using a program and explore some of its properties
Combinations and Pascal's triangle	Link combinations to the elements in Pascal's triangle
Binomial expansion	Expansion of brackets



The quincunx

Activity 8 Pascal's triangle

Aim: Generate Pascal's triangle as a spreadsheet and explore some of its properties.

Pascal's triangle has many patterns. It was originally developed by the Chinese. To generate:

- start with two 1's
- form the next row by putting 1's on the outside
- sum numbers that are adjacent to each other and write below

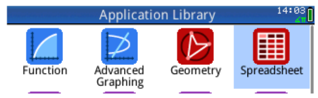
1. Fill in the missing values relating to Pascal's triangle and calculate the sum for each row.

Row #									Row sum
	1								
1	1								
2	1	2	1						
3	1		3	1					
4	1	4	6	4	1				
5			10			1			
6		6							
7								1	

Use the Prime Spreadsheet to create Pascal's triangle

Open Spreadsheet app.

- Press Apps and tap Spreadsheet

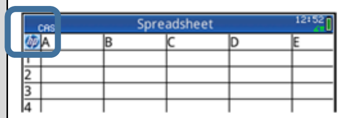


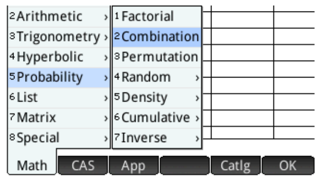
Insert formulae

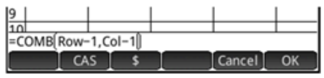
- Tap the upper-left corner to select the entire sheet
- Press Shift [≛] to start a new formula.
- Then press [toolbox]
  Tap Math > 5Probability > 3Combinations.

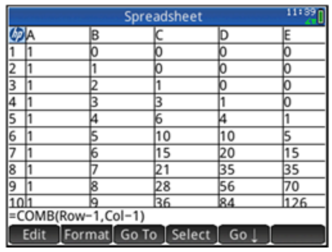
enter Row-1,Col-1, as shown to the right.

- Press Vars
- Tap App > 1Spreadsheet > 1Numeric > 3Row
- Enter –1,
- Select Col in a similar manner.
  Or
  You can always just type names in letter by letter, using ALPHA for uppercase and ALPHA Shift for lowercase letters.
- Tap OK to see the spreadsheet fill with Pascal's triangle! Use your finger to scroll through the spreadsheet.
- To clear the entire spreadsheet, tap on the upper-left corner and press Shift [≛]









Pascal's triangle has many interesting properties and patterns.

2. Use your spreadsheet to extend the triangle to the 12th row and sum the elements in each row.

Row #		Row sum
6	1 6 15 20 15 6 1	64
7	1 1	
8	1	
9	1	
10	1	
11	1	
12	1 12 66	

3. State:
    a) The third number in the 10^{th} row;

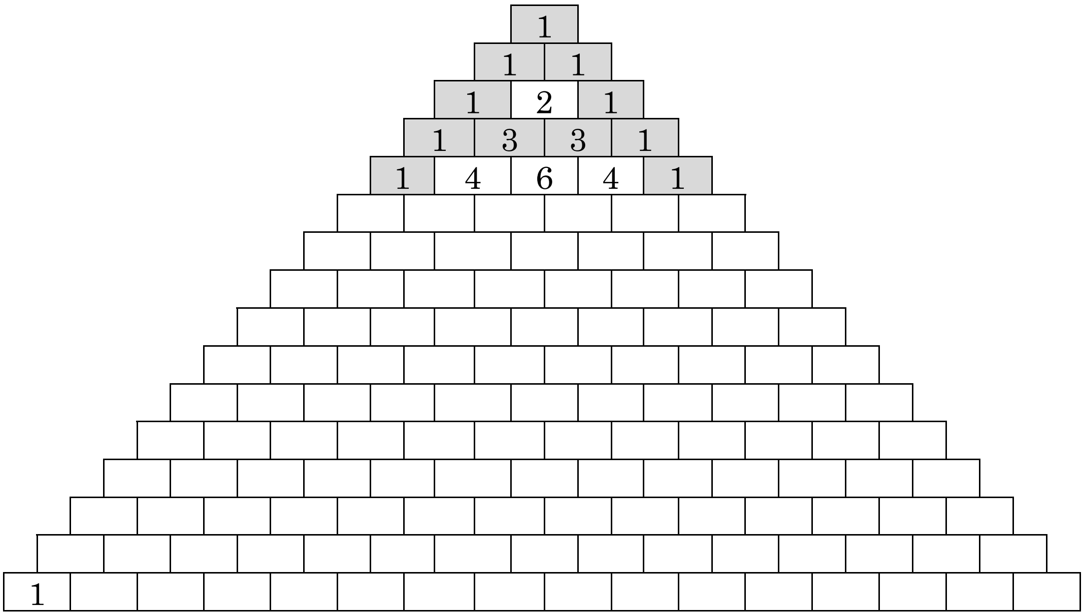
    b) The third last number in the 10^{th} row;

    c) The fourth number in the 15^{th} row; and

    d) The row and position of 78.

4. Describe how the sum in the next row is related to the sum in the previous row. Justify why this must always be so.

5. Colour all the spaces where the element in Pascal's triangle is odd.



Pascal's triangle is full of patterns. A quick search for *Pascal triangle pattern* will provide rich opportunities for exploration.

Activity 9 Combinations and Pascal's triangle

Aim: Explicitly calculate any element in Pascal's triangle.

1. A group of people meet and they each shake hands with each other exactly once.

   a) How many handshakes take place if:

      i) There are 4 people in the group; or

      ii) There are 7 in the group?

   b) What is the smallest group size where the number of handshakes is greater than 100?

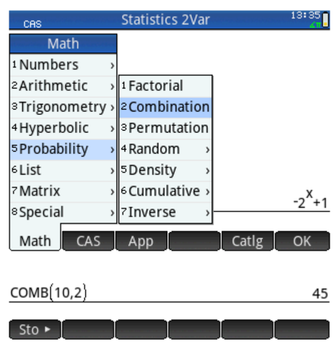
   The problem can be restated as
   in how many ways can two people be selected from the group.

2. Use Prime to calculate combinations

   $\binom{n}{r}$ or ${}^{n}C_{r}$ is the number of different ways of selecting r members from a group of size n. You may find it helpful to read this as **n choose r**.

Calculate value of a combination

- Press CAS Settings
- Press Mem B
- Tap Math, select
  Probability > Combination
- Complete entry
  e.g. ${}^{10}C_2$ is entered as COMB(10,2)



What is the value of:

a) $^{10}C_2$

b) $\binom{13}{2}$

c) $^{13}C_{11}$

d) $\binom{7}{3}$

e) $\binom{7}{3}+\binom{7}{4}$

f) $^{8}C_4$

g) $\binom{6}{6}$

h) $^{6}C_0$

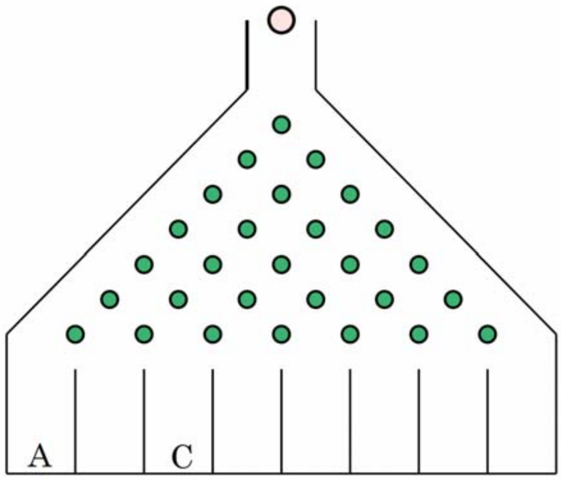
3.

a) Evaluate the following combinations to complete the table.

$\binom{4}{0}$	$\binom{4}{1}$	$\binom{4}{2}$	$\binom{4}{3}$	$\binom{4}{4}$
$\binom{5}{0}$	$\binom{5}{1}$	$\binom{5}{2}$	$\binom{5}{3}$	$\binom{5}{4}$
$\binom{6}{1}$	$\binom{6}{2}$	$\binom{6}{3}$	$\binom{6}{4}$	$\binom{6}{5}$
$\binom{7}{1}$	$\binom{7}{2}$	$\binom{7}{3}$	$\binom{7}{4}$	$\binom{7}{5}$

b) Describe how your results above are connected to Pascal's triangle?

4. The quincunx.



Balls are fed in at the top. They may either bounce left or right off each peg they hit. How many different ways are there for reaching each bin at the bottom?

a) How many different ways are there for the ball to end up in

i) Bin A; or

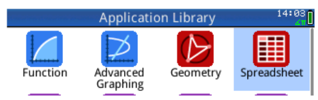
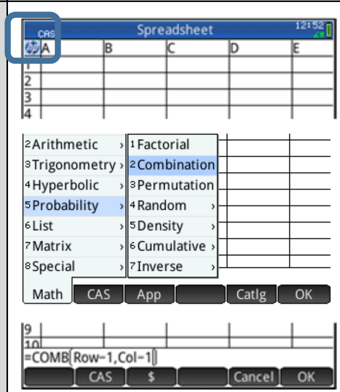
ii) Bin C?

A reason why combinations are connected to Pascal's triangle.

b) Another way of thinking of the problem:
The ball moves either left or right at each peg. How many moves right (or left) does the ball make to reach the bottom?

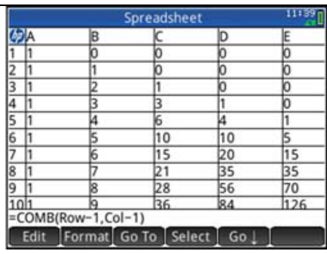
For bin A all the moves are left, i.e. 0 of the 7 moves are to the right.
For bin C there must be 5 moves left and 2 moves right. I.e. how many ways are there of choosing the two right (or 5 left) from the seven moves? Write these using combination notation.

Use the Prime Spreadsheet to create Pascal's triangle

Open Spreadsheet app. • Press [Apps Info] and tap Spreadsheet	
Insert formulae • Tap the upper-left corner to select the entire sheet • Press [Shift] [:=] to start a new formula. • Then press [Mem B] Tap Math > 5Probability > 3Combinations. **enter Row-1,Col-1**, as shown to the right. • Press [Vars Chars A] • Tap [App] > 1Spreadsheet > 1Numeric > 3Row • Enter –1, • Select Col in a similar manner. Or You can always just type names in letter by	

letter, using ALPHA for uppercase and ALPHA Shift for lowercase letters.

- Tap OK to see the spreadsheet fill with Pascal's triangle! Use your finger to scroll through the spreadsheet.
- To clear the entire spreadsheet, tap on the upper-left corner and press Shift [clear key]



5. Use Prime to calculate:

   a) The fourth element in the 20th row of Pascal's triangle;

   b) The largest element in the 25th row; and

   c) The first element over 100 in the 13th row.

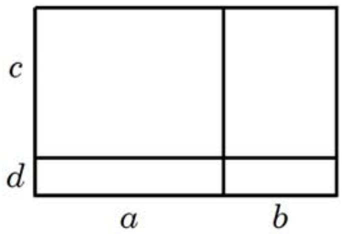
Learning notes

The purpose in this activity is for you to see the connection between combinations and Pascal's triangle. Can you explain why the connection exists?

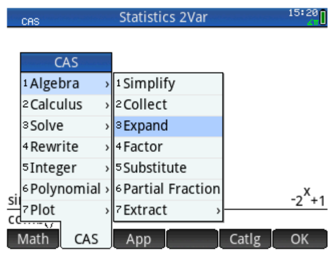
Activity 10 Binomial expansion

Aim: Understand expanding products of brackets.

1. Marcia uses an area model to explain why $(a+b)(c+d) = ac + ad + bc + bd$.
   She begins with a diagram and the statement that the area of the large rectangle is the same as the sum of the four small rectangles.
   Complete Marcia's argument.



2. Use CAS to expand expressions

Expand expressions: • Press CAS Settings • Press Mem B select **Algebra > Expand**	
• Enter expression and press Enter	

a) Expand each expression and record the number of terms:

i) $(a+b+c)(x+y)$

ii) $(a+b+c+d)(x+y)$

iii) $(a+b+c)(x+y+z)$

iv) $(a+b+c+d+e)(x+y+z)$

b) How many terms are to be expected when a bracket of m terms is multiplied by a bracket of n terms?

c) Justify your answer

d)

i) Expand $(a+b)(a+b+c)$

ii) How many terms are there?

iii) Reconcile this result with your earlier answer.

3. More than two brackets

a) Expand each expression and record the number of terms:

i) $(a+b)(m+n)(x+y)$

ii) $(a+b+c)(m+n)(x+y)$

iii) $(a+b+c)(m+n)(x+y+z)$

iv) $(a+b+c+d)(m+n)(x+y)$

v) $(a+b)(c+d)(m+n)(x+y)$

b) How many terms are to be expected when brackets are expanded?

c) Justify your answer.

4. Binomial powers

a) Expand the following using Prime. Record your answers with the terms ordered with decreasing powers of a.

Expression	Expansion
$(a+b)^2$	
$(a+b)^3$	$a^3+3a^2b+3ab^2+b^3$
$(a+b)^4$	
$(a+b)^5$	
$(a+b)^6$	

b) Use your answer to a) to write the coefficients of each term in a triangle pattern. This has been started for you.

Expression

$(a+b)^2$ 1 2

$(a+b)^3$ 1 3 3 1

$(a+b)^4$

$(a+b)^5$

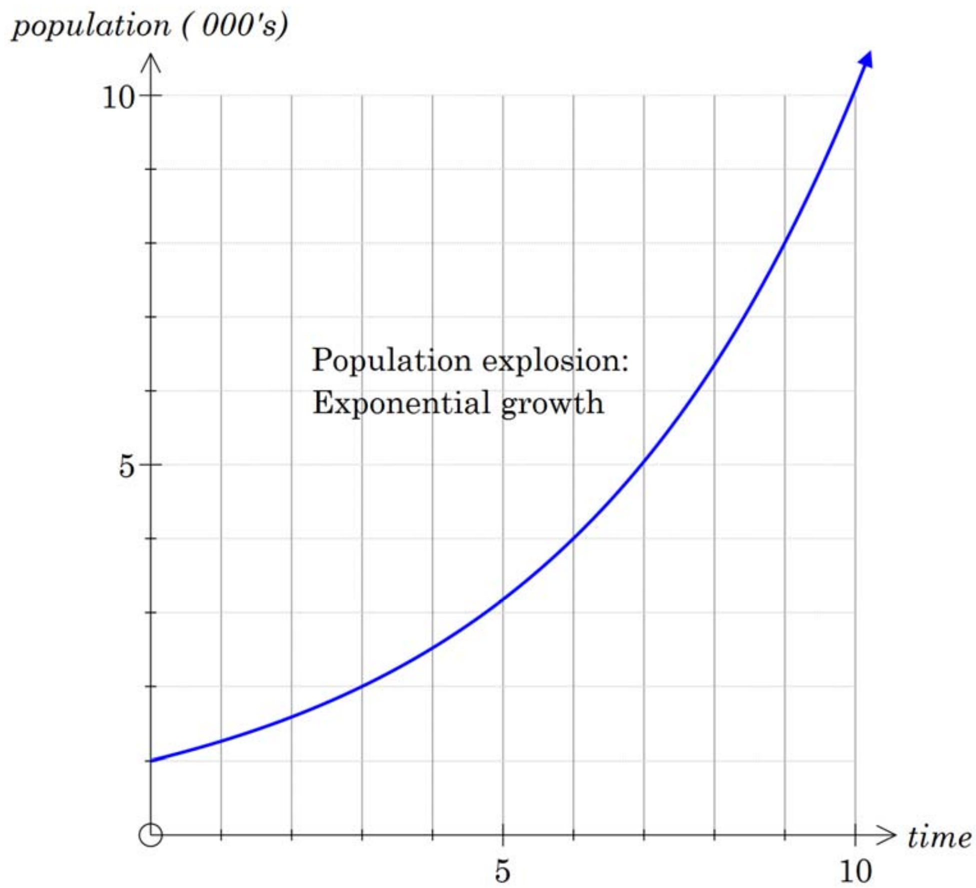
$(a+b)^6$

c) What is the connection with Pascal's triangle?

5. Expand $(2x^2-5)^3$ without the use of a calculator.
(Hint: Use Q4 a) and then simplify)

Chapter 4 Exponentials

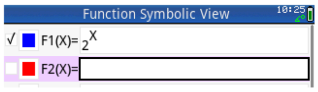
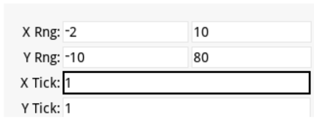
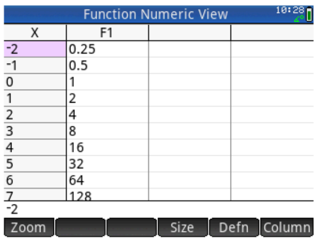
Investigation	Key concepts
Exponential functions	Key features of exponential functions
Exponential equations	Solve exponential equations
Index laws	Simplify expressions and identify the rules used.
Scientific Notation	Entry and display of numbers in scientific notation
Carbon dating	Application of exponential function to model decay processes



Activity 11 Exponential Functions

Aim: Graph exponential functions and identify key features.

1. Graph the function $y = 2^x$

Press [Apps Info] and tap **Function** Enter the function $y = 2^x$	
Set the graph window to match the grid • Press [Shift] [Plot Setup] • Enter values as shown	
Display the graph • Press [Plot Setup]	
Display table of values • Press [Num Setup]	

a) Complete the tables of values for $y = 2^x$

x	0	2	6	10
$y = 2^x$				

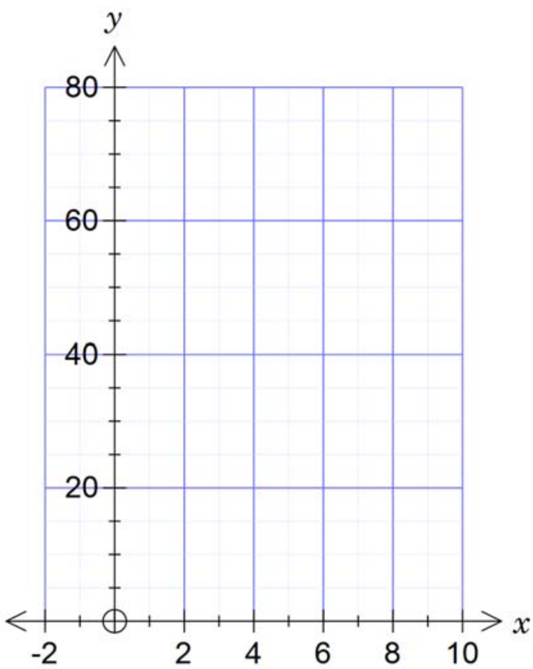
x	-2	-1	1.2	1.5
$y = 2^x$				

b) What happens to the value of y as

i) $x \to \infty$

ii) $x \to -\infty$

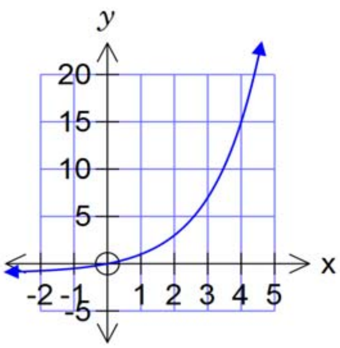
c) Sketch the graph of $y = 2^x$



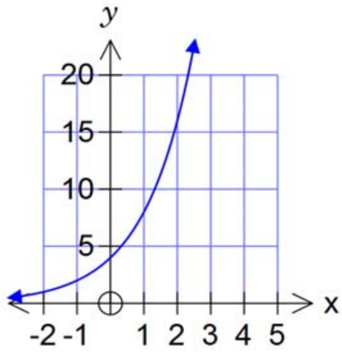
2. Mix and match the equation with the corresponding graph and key features by completing the table on the next page.

Equation A	**Equation B**	**Equation C**
$y = 8 - 2^x$	$y = 2^x - 4$	$y = 2^{x+2}$
Equation D	**Equation E**	**Equation F**
$y = 2^x - 1$	$y = 2^{-x}$	$y = 2^{x-2}$

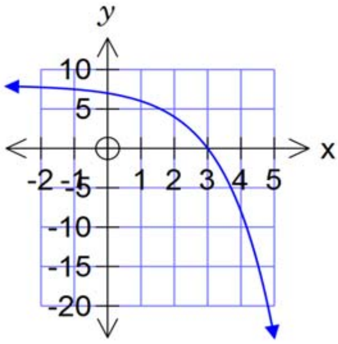
Graph I



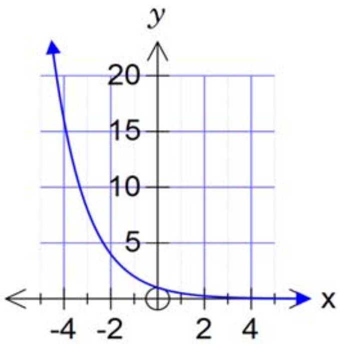
Graph II



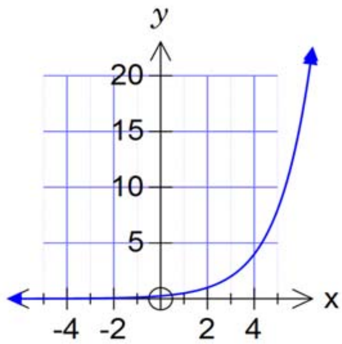
Graph III



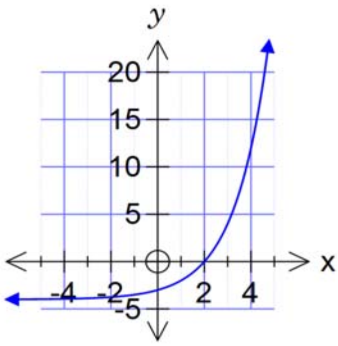
Graph IV



Graph V



Graph VI



Key features 1	**Key features 2**	**Key features 3**
as $x \to \infty, y \to \infty$ as $x \to -\infty, y \to 0$ intercepts: $(0,4)$	as $x \to \infty, y \to 0$ as $x \to -\infty, y \to \infty$ intercept: $(0,1)$	as $x \to \infty, y \to \infty$ as $x \to -\infty, y \to 0$ intercept: $(0,0.25)$
Key features 4	**Key features 5**	**Key features 6**
as $x \to \infty, y \to \infty$ as $x \to -\infty, y \to -4$ intercepts $(0,-3)$ & $(2,0)$	as $x \to \infty, y \to -\infty$ as $x \to -\infty, y \to 8$ intercepts: $(0,7)$ & $(3,0)$	as $x \to \infty, y \to \infty$ as $x \to -\infty, y \to -1$ intercepts: $(0,0)$

a) Write the number of the corresponding Key features and Graph to each equation.

Equation	Key features	Graph
A		
B		
C		
D		
E		
F		

b) State the equation of the horizontal asymptote

Equation of asymptote

3. Summarise your findings from Q2 by completing the following statements for the function $y = 2^{x-b} + c$.

a) As $x \to \infty$, $y \to$

b) As $x \to -\infty$, $y \to$

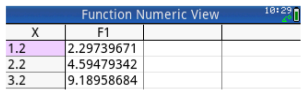
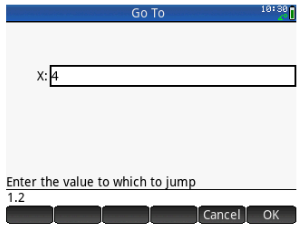
c) The equation of the horizontal asymptote is ________________.

d) The y-intercept is ____________.

e) When there is an x-intercept the value of c is ____________________ .

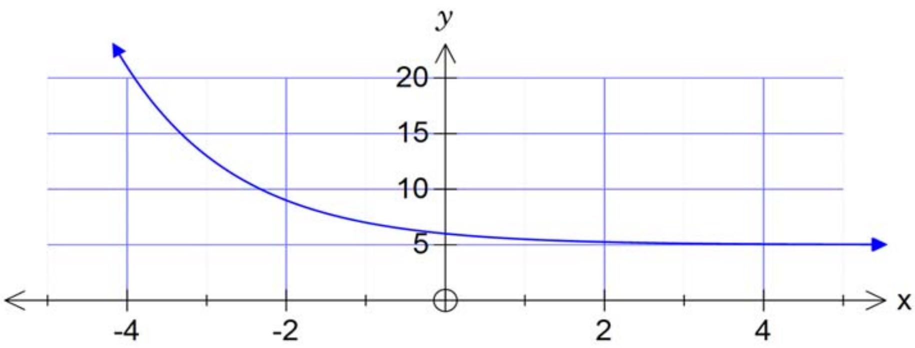
Learning Notes

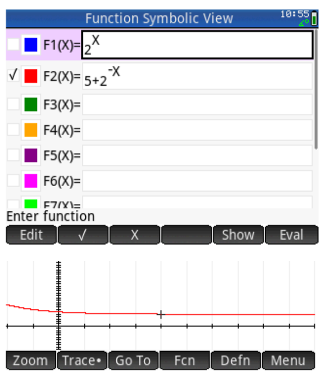
Q1 a) To calculate the y-value for the table

Press Num Setup enter the x-value and press Enter	
Or • Press Plot Setup and Go To • Enter the x-value • Press Enter	

Q2 You may begin by drawing the graph. As you are doing the mix and match look for connections such as what is it in the equation that leads to differences in the graphs and key features.

When describing behaviour near an asymptote it is useful to add the direction that the graph is approaching the asymptote from. E.g. in this graph $y = 5 + 2^{-x}$ it appears that as $x \to \infty$, $y \to 5^{+}$ (y is approaching 5 from above).



Behaviour as $x \to \pm\infty$ • Press Plot Setup tap Menu and tap Trace • Look at how the y-value is changing Or • Substitute appropriate small or large values for x	

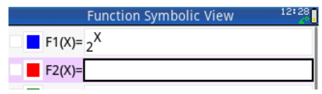
There are also important links to be made with transformations of functions. I.e. what is required to reflect the graph in the x and y-axes and translate the graph.

Activity 12 Solve exponential equations

Aim: Solve exponential equations graphically and using CAS.

1. Solve $2^x = 3$ for x.

Draw the graph of $y = 2^x$ in Function App



Use your graph to determine the solution in the following ways:

a) x lies between which two consecutive whole numbers?
(Use the table of values)

b) Using Trace, what is the x-value that gives y closest to 3?

c) Use the intersection of the graphs $y=3$ and $y=2^x$ to determine x, correct to 4 decimal places.

2. Find solutions (3 decimal places where necessary) to the following equations:

a) $2^x = 8$

b) $2^x = 100$

c) $2^x = 1024$

d) $3^x = 729$

e) $5^x = 5942$

3. Complete the quiz

- Use your Prime to work out each question.
- Round decimal answers to 3 decimal places.
- Sum your answers and compare to the given total.

	Question	Hint	Answer
a)	Simplify $2^{n+2} - 5\times 2^{n} + 1$	CAS Settings Mem B **1Algebra > 1Simplify**	
b)	Solve $x^{2.5} = 32$	CAS Settings Mem B **3Solve > 1Solve**, x	
c)	Solve $x^{1.5} = 27$		
d)	Solve $y^{-1} = \frac{1}{8}$	Make sure you are solving for *y*	
e)	Solve $2^{x} = 33$		
f)	Solve $3\times 2^{x} = 99$		
g)	Solve $\frac{3\times 2^{x}}{11} + 1 = 10$		
h)	Solve $49^{2x-1} = 7$		
i)	Simplify $\left(\frac{3^{n+3} - 3^{n}}{3^{n+1} - 3^{n}}\right)$		
j)	Evaluate $\frac{3^{1.7} - 2^{3.1}}{5^{-0.8} + 1.1}$		
	Total Q's a) – j)		$49.355 - 2^{n}$

EXTENSION
Which questions are you able to do without a calculator?

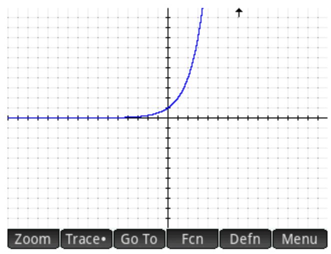
Learning Notes

Q1 b) Using Trace

With the function graph window active

- Tap Menu then TRACE•
- Use the arrow keys to move along the curve

The values are displayed in the bottom of the window.



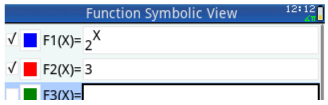
Q1 c) Find the intersection of the graphs $y=3$ and $y=2^x$

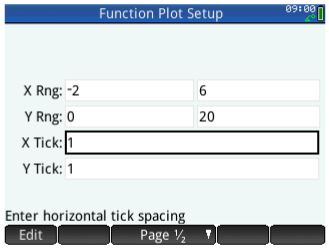
Draw the line y**=3**

- Tap Symb Symb Setup
- Enter 3 for y2 and press Enter

Adjust the view window

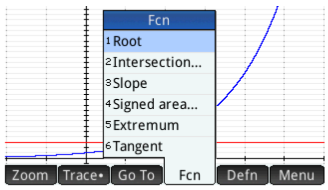
- Press Shift Plot Setup
- Set ymin to 0 and ymax to 20 or other appropriate values for the problem
- Press Plot Setup





Find the point of intersection

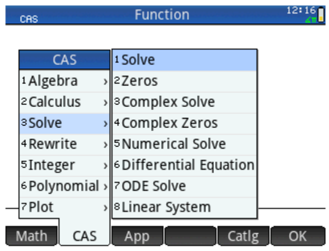
- Tap Fcn Select **2Intersection**

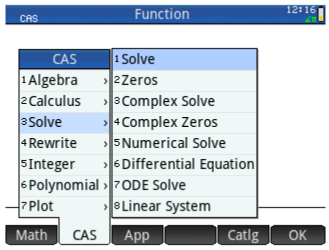


Solve with CAS

Solve equations

- Press CAS Settings Mem **3Solve > 1Solve**
- Enter the equation , x
- Enter





Activity 13 Index laws

Aim: Use Prime to work efficiently with indices.

Set Prime to CAS mode.

Enter each expression in Prime, record the output and complete the table.

Expression	Prime display	Rule(s) used by CAS
1. 2^{-4}		
2. $\left(\frac{2}{3}\right)^{-1}$		
3. $a^0 + 2b^0$		
4. c^{-3}		
5. $\left(2c^3\right)^{-2}$		
6. $\left(\frac{5}{7}\right)^{-3}$		
7. $\frac{4^3 \times 2^5}{2^9}$		
8. $5^3 \times 5^{-7} \times 5^4$		
9. $\frac{3^2}{3^{-2}}$		
10. $\frac{d^{-3}}{d^2}$		
11. $\text{solve}\left(2^x = \frac{1}{32}\right)$		
12. $\text{solve}\left(2^{2x-1} = \frac{1}{32}\right)$		

Learning notes

For the right hand column you may refer to the following list of index laws.

$a^m \times a^n = a^{m+n}$	Multiplying powers with the same base
$\dfrac{a^m}{a^n} = a^{m-n}$	Divide powers with the same base
$a^0 = 1, \; a \neq 0$	To the power zero
$a^{-n} = \dfrac{1}{a^n}$	Negative power
$\left(a^m\right)^n = a^{mn}$	Power of a power
$(ab)^n = a^n b^n$	Power of a product
$\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$	Power of a quotient
$a^{\frac{m}{n}} = \sqrt[n]{a^m} \text{ or } \left(\sqrt[n]{a}\right)^m$	Fractional indices

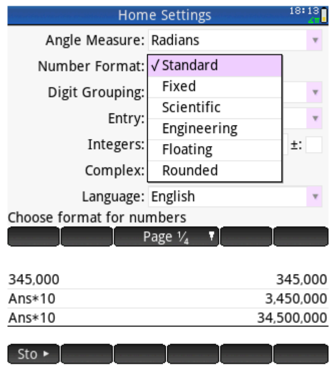
Activity 14 Scientific notation

Aim: Understand scientific notation and representation on Prime.

1. Complete the table.

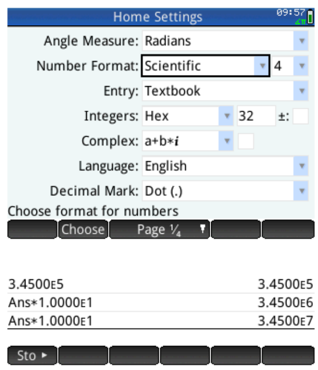
Set up

- Press [Settings] to select the home screen
- Press [Shift] [Settings] to open home settings window
- Ensure Prime is in **Standard** mode.
- Press [Settings]
  Enter 345 000 and press [Enter]
- Multiply the result by 10
  - o press [×] and ans× will appear
  - o enter 10 and press [Enter] .



Change to scientific mode

- Press [Shift] [Settings]
- Ensure Prime is in **Scientific** mode.
- Press [Settings] to see results displayed in scientific notation



Input	Prime output (Standard format)	Prime output (Scientific format)	Scientific notation
345 000	345000	3.45E5	3.45×10^5
ans [×] 10	3450000		
ans [×] 10			
ans [×] 10			
ans [×] 10			
ans [×] 10			
ans [×] 10			

2. Complete the table.

• Ensure you are in Standard mode
• Enter 34.5 and press [Enter]
• Divide the result by 10
  o Press [÷] and ans [÷] will appear
  o enter 10 and press [Enter] .
• Repeat and use the results to fill in the table below	

input	Prime output	Decimal number	Scientific notation
34.5	34.5	34.5	3.45×10^1
Ans [÷] 10	3.45		
Ans [÷] 10			
Ans [÷] 10			
Ans [÷] 10			
Ans [÷] 10			
Ans [÷] 10			

3. How does Prime display numbers in scientific notation?

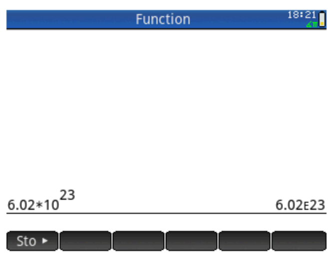
Enter numbers in scientific notation

e.g. 6.02×10^{23}

- Enter 6.02
- Press [EEX] enter 23 Press [Enter]

Or

- Enter 6.02
- Press [×]
- enter 10, tap [x^y] , enter 23 and press [Enter]



4. Evaluate the following expressions. Round to three significant figures and write in scientific notation.

   a) $3.00\times10^{8}\times\left(1.47\times10^{-17}\right)^{2}$

   b) $\sqrt[3]{6.02\times10^{23}}$

5. Calculate the

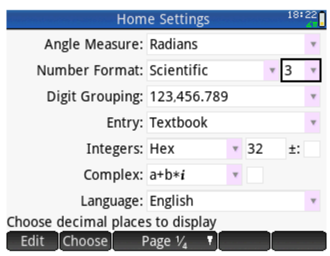
   a) number of spare electrons on a statically charged object carrying -1.28×10^{-11} Coulombs of charge, rounded to 3 significant figures. (Each electron has a charge of -1.602×10^{-19} Coulombs)

   b) mass of the Earth based upon a sphere of radius 6.378×10^{6} m and average density of 5.513g/cm^3, rounded to 2 significant figures.

Learning Notes

The number format of the Prime can be be changed to force the output to be rounded to a given accuracy and to output answers in scientific notation.

- Press Shift Settings
- Changing Number Format to Scientific 3 forces Prime to output answers in scientific notation rounded to 3 decimal places (4 significant figures)



Activity 15 Carbon dating

Aim: Model decay processes with exponential functions.

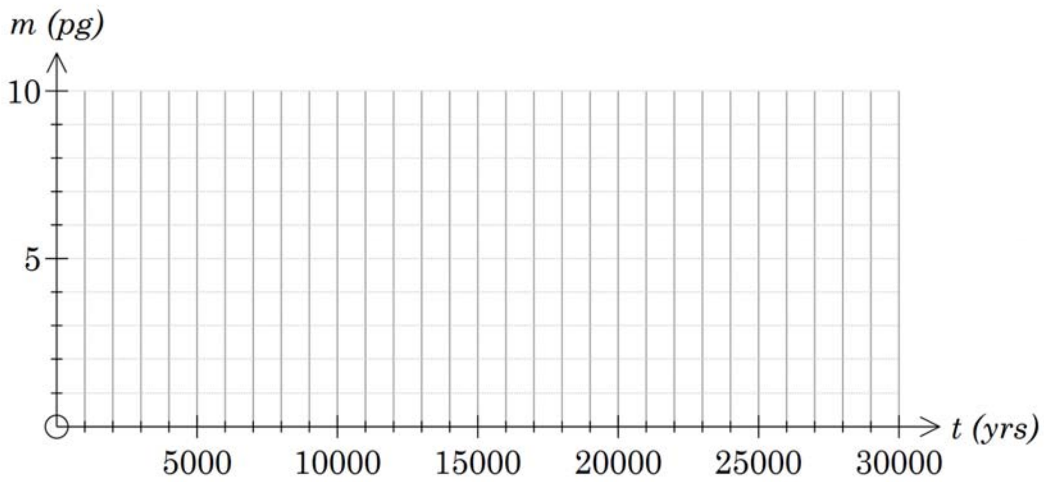
Radioactive materials break down over time. The time taken for half of the material to decay is the half-life and is constant. The amount remaining is given by the equation $W = W_o 2^{-\frac{t}{k}}$ where W_0 is the original amount, t is the elapsed time and k is the half-life.

Trees are made of wood. When new wood is grown, the tree uses Carbon from the atmosphere, a small percentage of which is radioactive Carbon 14 (C14). Over time the C14 breaks down into non-radioactive Carbon 12 (C12) with a half-life of 5720 years. This knowledge can be used to date old wood and charcoal from campsites.

1. A sample contains 8.8×10^{-12} g of C14 (8.8 picograms pg).

   a) Write an equation for the weight of C14 remaining after t years.

   b) Draw a graph of this model for $0 \leq t \leq 30000$



   c) For this sample determine the weight of C14 after:

      i) 130 years

      ii) 3000 years

      iii) 15000 years

d) How many years before there is less than:

i) 10% remaining?

ii) 0.1% remaining?

When dating old objects the original amount is not known. An initial approach is to assume the ratio of C14:C12 = 10^{-12} and has been constant over time.

2.

a) Explain why, under this model, the original amount of C14 in picograms equals the amount of C12 in grams. (1 pg = 10^{-12} g)

b) Complete the table using the model.

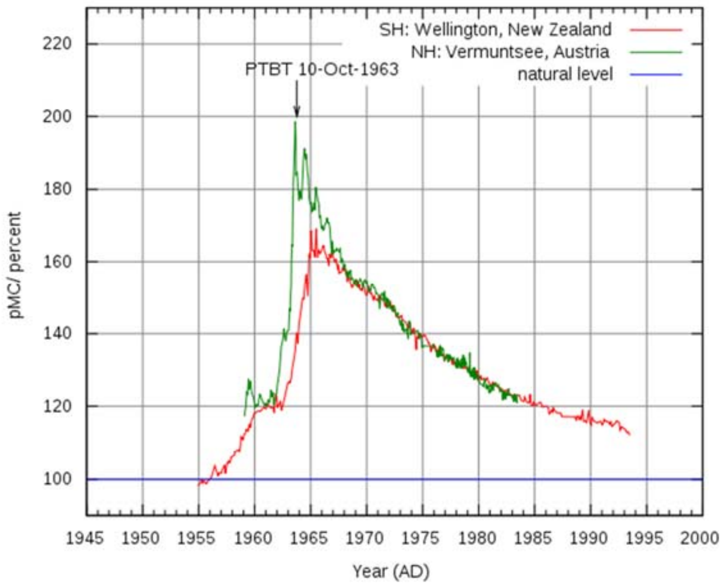
Sample	C14 (pg)	C12 (g)	C14 (pg) when carbon was fixed	Age
Charcoal	1	2	2	5720
Tree	0.38	0.42	0.42	
Peat	0.0127	0.063		
Bone	6.98×10^{-3}	7.18×10^{-3}		
Tooth	4.93×10^{-3}	0.0061		

The nuclear tests in the 1950s and '60s produced C14 with the result that the concentration of C14 in the atmosphere effectively doubled. This has made it possible to date, and to help identify, human remains found after accidents and natural disasters.

The graph over the page shows the concentration of C14 in the atmosphere since the atomic tests as a percentage of the long term average. The shape of the curve suggests an exponential decay.

3. Post 1965

http://en.wikipedia.org/wiki/Radiocarbon_dating#The_effects_of_human_activity



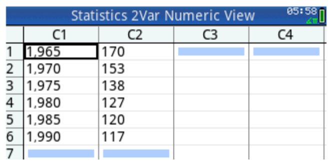
Year	C14 (% of long term average)
1965	170
1970	153
1975	138
1980	127
1985	120
1990	117

Table of values generated from the graph.

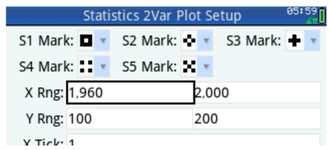
a)

Generate a model for carbon dating post 1965 using the data in the table above

- Enter data in Statistics 2VAR



- Draw the graph
- Adjust the scales to get a good fit with an exponential regression (see Learning notes for detailed instructions)



b) Use your model to date the objects.

Sample	C14 (pg)	C12 (g)	C14 (% of long term atmospheric concentration)	Age (year)
Bone	9.83×10^{-3}	7.18×10^{-3}	137%	
Tooth	4.73×10^{-3}	0.0029		
Bone	0.027	0.021		

4. Calculate dates for samples with:

a) 0.16 pg C14 and 0.13 g C12

b) 0.16 pg C14 and 0.19 g C12.

EXTENSION

Does the decay of C14 affect the accuracy of post 1965 carbon dating using the model developed in Q3?

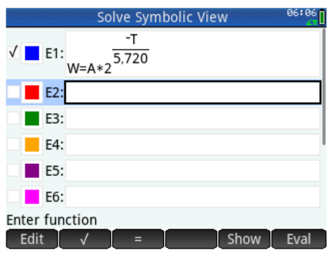
How accurate are the measurements and subsequent calculations?

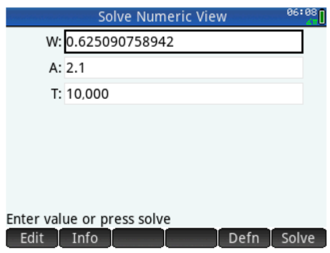
Learning Notes

Q1 It may be easiest use Solve App to evaluate part c) and solve equations in d)

Solve App

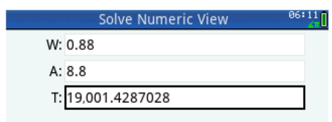
- ALPHA
- Enter the equation and press Enter Note W_0 is defined as A
- Press Shift and enter the known values.
- Toggle to W and Tap Solve





Solve for *t*

- Repeat above steps to solve for t



Q3 Generate a model using an exponential regression.

Enter data in Statistics

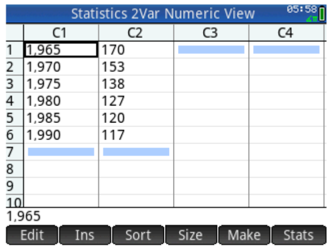
- [ALPHA] Statistics VAR 2
- Enter years in C1 and percentages in C2

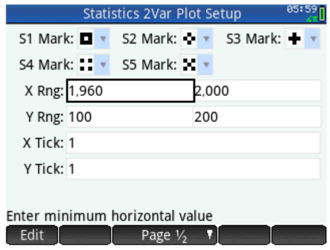
Set Graph parameters to draw scatterplot

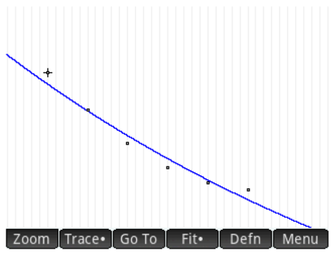
- Press [Shift] [Plot]
- Set parameters as per screen shot to right

Draw the graph

- Press [Plot]





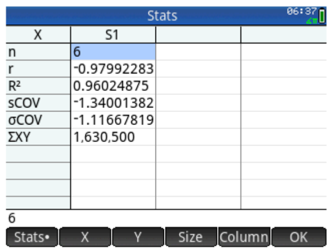


Fit the curve

- Tap [Fit•]

To Calculate the regression

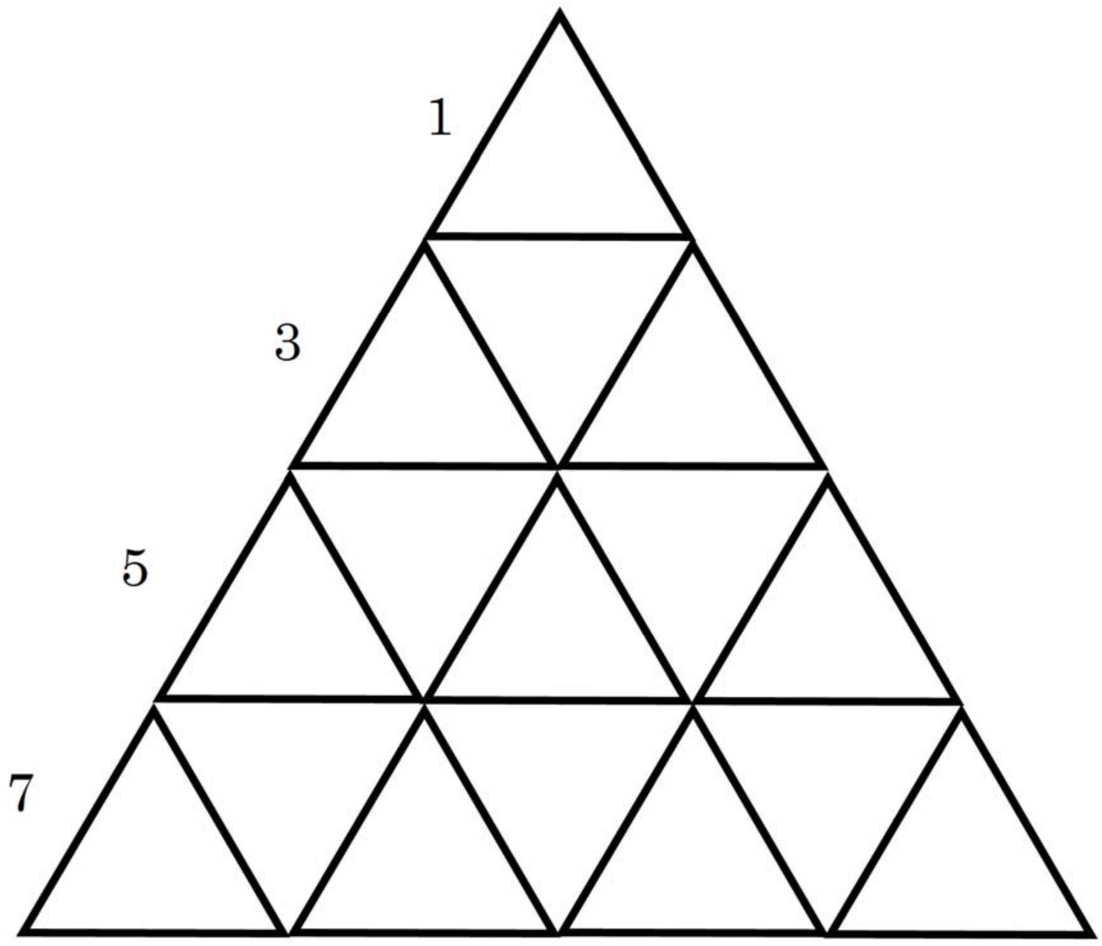
- Press [Num] tap [Stats] and read the correlation co efficient
- Press [Shift] and [Show] to see the equation



$$1+3+5+7+\ldots+(2n-1)=n^2$$

Chapter 5 Sequences and series

Investigation	Key concepts
Rolls of tape and towels	Sum of an arithmetic sequence
Paper folding	Geometric sequences



$$1+3+5+7+\ldots+(2n-1)=n^2$$

Activity 16 Masking tape

Aim: Investigate growth using iterative methods.

Items such as masking tape, toilet paper and electrical tape are sold as rolls. As the roll is wound, each layer can be modelled as a circle with the diameter of each circle increasing by twice the thickness of the tape.

Consider a roll of sticky tape with internal diameter 3.5 cm (the external diameter of the cardboard spool) and thickness 0.05 mm.

1. Complete the table

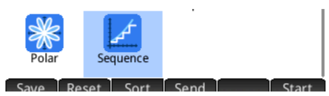
Winding	Diameter (mm)	Length of winding (mm) (πD)	Total length wound (mm)
1	35	110	110
2	35.1	110.3	220.3
3	35.2		
4			
5			

2. Explain why the next winding is 0.1π longer than the previous winding.

3. Duplicate the results from Q1 using the sequence application.

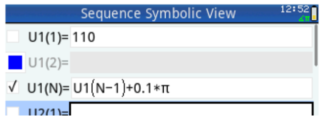
Set up sequence app

- Select Sequence App



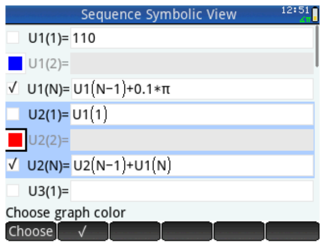
Enter recursive formula

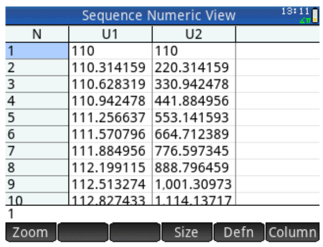
- Enter 110 for U1
- Tap U1(N) and type formula U1(N–1)+0.1×π
- Press [Enter]



Show the series sum

- In U2(1) Enter first term
- Tap U2(N) and enter formula U2(N–1)+U1(N)
- Press [Num Setup] to see series and sum





Use the sequence values to determine:

a) How long is the 100^{th} winding?

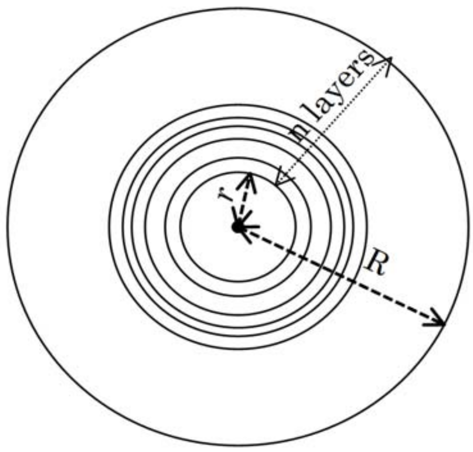
b) What is the total length of tape after 100 windings?

c) How many windings are required for a 20 metre roll?

Hint: You may need to change the domain.

d) How long the tape will be if the tape is half the thickness but the complete roll is the same size as the 20 metre roll in c).

4. The roll of paper towels below has 84 sheets of size 279 mm × 279 mm.



Marcel measures the radius of the full roll (R) at 67 mm and the radius of the cardboard centre (r) as 17 mm.

a) Unrolled, what is the length of paper (L)?

b) Justify why $nt = 50$ mm where n is the number of layers and t is the thickness of each layer.

c) Determine the average thickness using a trial and error approach. (See Learning notes for instructions)

d) Solve the problem using an algebraic approach.

i) Show that $23436 = 84\pi n$

The sum of an arithmetic series is $S_n = \frac{n}{2}(a + l)$ where n is the number of terms, a the first term and l the last term.

ii) Determine the thickness of the paper towels.

Learning Notes

Q3 To determine length of 100th winding

Press [Num Setup] for table view of the sequence

With cursor in column N enter 100 and press [Enter]

Instructions for Q4 c)

Calculations to set up the sequence

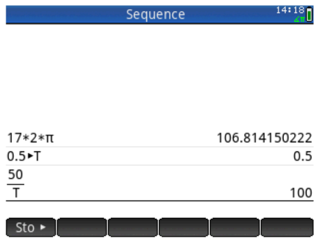
- Press [Settings]
- Enter $17 \times 2\pi$ to calculate the circumference of the first layer

Store a value for the thickness

- 0.5 [Sto ▸] T

Calculate the number of windings

- Enter (50 / T)



Sequences with a constant difference are called arithmetic sequences. The following formula is useful for answering Q4 d).

The sum S_n of an arithmetic series is

$$S_n = \frac{n}{2}(a+l) \text{ or } S_n = \frac{n}{2}(2a+(n-1)d)$$

where n is the number of terms,
a the first term,
l the last term and
d the constant difference between successive terms.

Activity 17 Paper and rice

Aim: Solve problems involving geometric sequences.

1. A very large sheet of cardboard measures 10 m by 10 m and is 0.5 mm thick. It is cut in half and one half is then placed on top of the other.

   a) Complete the table

Cut	Base	Height of stack
0	10 m × 10 m	0.5 mm
1	10 m × 5 m	1 mm
2	5 m × 5 m	2 mm
3	5 m × 2.5 m	
4		
5		
6		

   b) Write a recursive formula for the height of the stack and enter this in Prime Sequence application.

   c) Write an explicit formula for the height of the stack in terms of the number of cuts.

   d) This process continues until the stack is 2 m high. How many cuts are required?

2. The emperor is so pleased with the sage who has rid his kingdom of pestilence that he offers a reward of the sage's choosing. Eventually the sage asks for one grain of rice on the first square of a chessboard and then double the number on each subsequent square.
http://en.wikipedia.org/wiki/Ambalappuzha

a) Complete the table

Square	Grains of rice G_n	Total number of grains T_n
1	1	1
2	2	3
3	4	7
4	8	
5		
6		

b) Write a recursive formulae for G_n.

c) Enter the recursive sequence for G_n in Prime and use the sum feature to duplicate the table above.

i) Which is the first square to require at least 1 cup of rice?

Properties of rice
1 grain weighs approximately 25 mg
7000 grains per cup (250 mL)

ii) By which square will the total amount be at least 1 bag (20kg)?

d) Write an explicit formulae for G_n.

e) Describe a container that would hold all the rice up to and including the

i) 31st square

ii) 45th square.

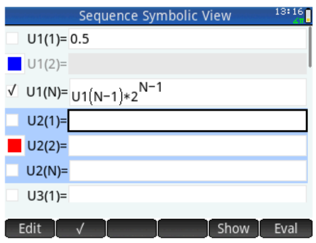
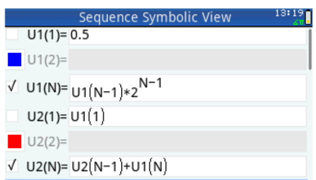
f) For T_n write

i) a recursive formula

ii) an explicit formula

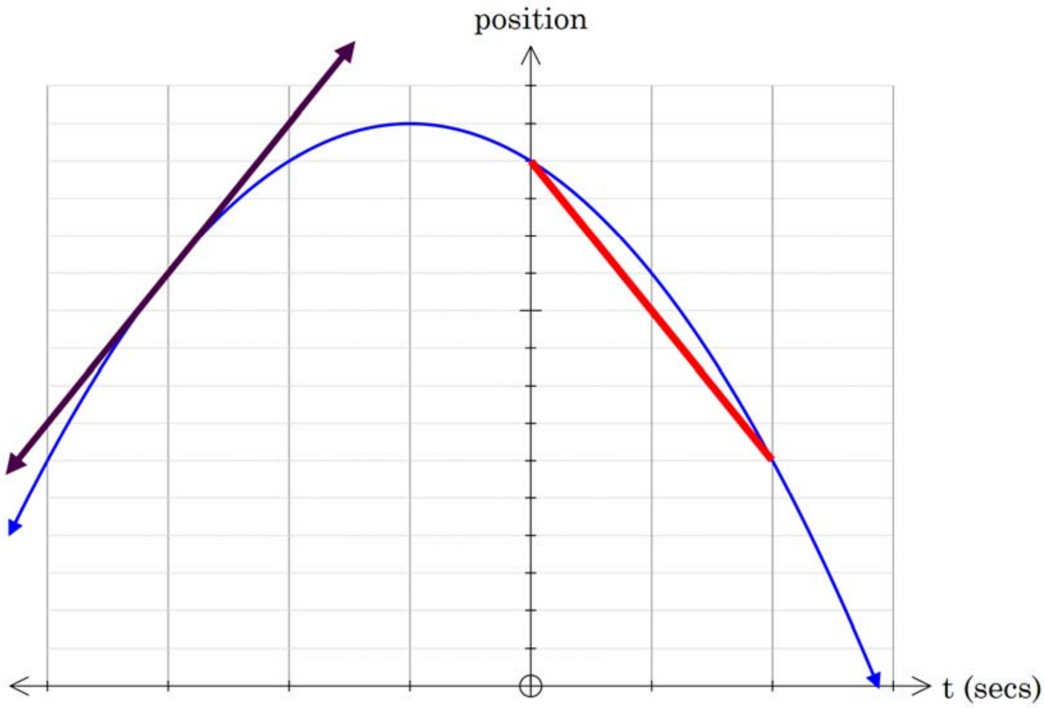
Learning Notes

Sequences with a constant ratio between successive terms are called geometric sequences. The sequences in this activity are geometric as there is a constant multiplier between successive terms.

Set up sequence app • Open Sequence • In U1(1) enter first term 0.5 **Enter recursive formula** • Tap U1(N) and enter formula U1(N-1) X 2^{N-1}	
Show the series sum • In U2(1) Enter first term ie and tap U2(N) and enter formula U2(N-1)+U1(N) • Press Num Setup to see series and sum	

Chapter 6 Differential calculus

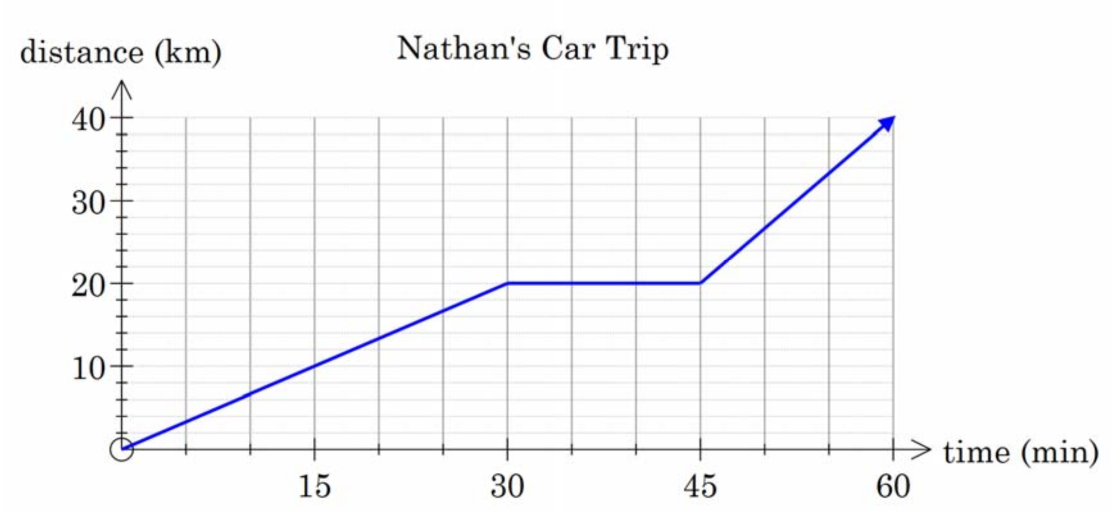
Investigation	Key concepts
Average speed	The gradient of a chord on a distance time graph is the average speed
Speed at an instant	Informally look at instantaneous speed during acceleration
Gradient of a tangent	Numerically investigate gradient of a tangent to a curve
Gradient functions	Sketch curves and relate key features of a function with its derivative function.
Differentiate	Compute derivatives
Tangents	Equation of tangents
Modelling motion	Application of differential calculus to rectilinear motion



Activity 18 Average speed

Aim: Understand that the gradient of a chord on a distance time graph is the average speed.

1. Every 15 minutes Nathan noted the distance travelled on his trip meter as he began his holiday trip. He has used this information to plot the graph.



a) Complete the table to show the measurements Nathan has used to create the graph. He left home at 5pm.

Time	Distance from home (km)
5:00	
5:15	
5:30	
5:45	
6:00	

b) What was Nathan's average speed in (km/h) for:

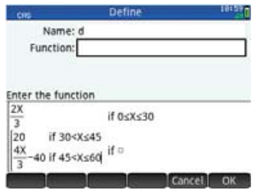
i) the first 30 minutes?

ii) the time interval between 45 and 60 minutes?

iii) the whole journey?

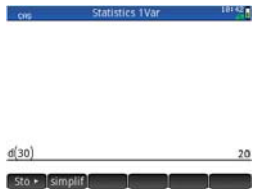
c) Estimate his average speed between 40 and 50 minutes.

2. Define Nathan's journey as a piece-wise function

Define the function	
• Press Shift $xt\theta n$ (Define) • Name the function d • Tap OK • Press Units and select disjoint function. • Toggle to bottom function and press Units again and select disjoint function to get 3 pieces to the function • Complete the entry as shown	

Check that the function gives the correct values for distance travelled, i.e. calculate $d(0)$, $d(15)$, $d(30)$, $d(45)$, $d(60)$ and compare to Q1 a).

a) According to this function how far has Nathan travelled at:

Calculate value of distance function	
• Press CAS (Settings) • Enter function name and value e.g. d(30) to calculate distance from home at 5:30	

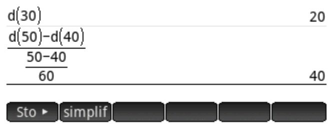
i) 5:06

ii) 5:40

iii) 5:50

iv) 6:15

b) Calculate Nathan's average speed (according to the function) between:

Calculate average speed E.g. in km/h between 5:40 and 5:50 • Enter the expression as shown • Edit values to recalculate	
Recalculate • Highlight expression • Tap Copy and edit as required	

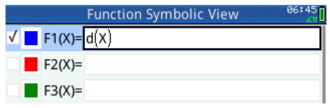
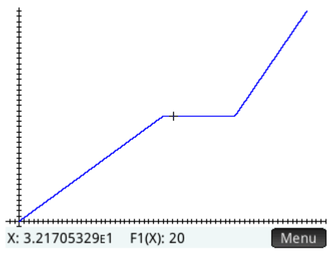
i) 5:06 and 5:40

ii) 5:42 and 5:55

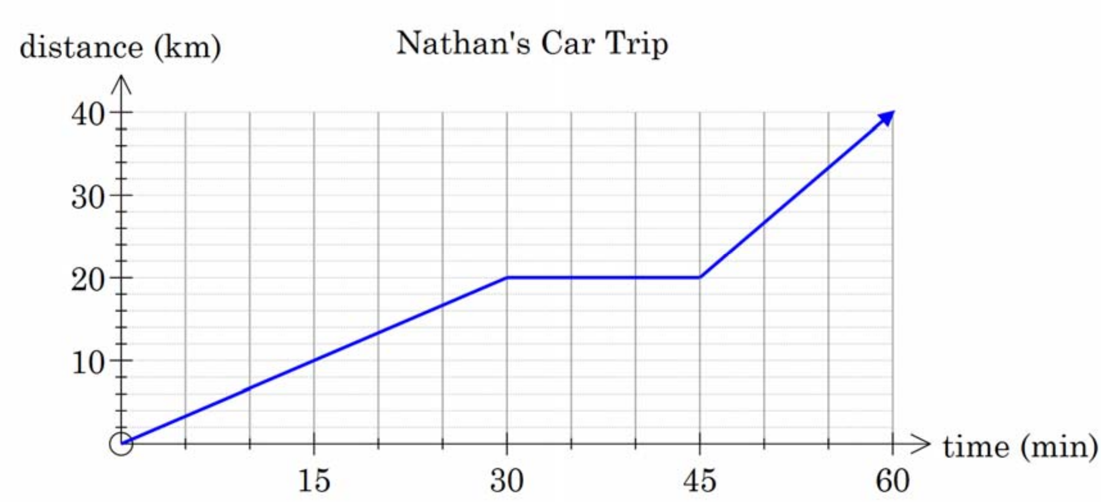
iii) 5:23 and 5:33

c) Explain the expression from the screenshot in part b) starting from the formula $\text{Average speed} = \dfrac{\text{distance travelled}}{\text{time taken}}$

d) Draw the graph d(X) on Prime.

Set up the graph • Press [Apps/Info] and function and enter d(X) into F1(X) press [Enter]	
Draw graph • Press [Plot/Setup] and pinch and pull to get the right screen size to see the graph OR Press [Shift] [Plot/Setup] to set the view window to match the grid	

e) Draw a line on the graph below to represent Nathan's trip between 5:40 and 5:50 if he had travelled at constant speed and the graph shows his correct distance from home at 5:40 and 5:50.



f) For the equation of the line drawn in d),

i) What is the gradient?

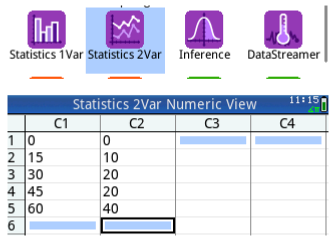
ii) What is the equation?

g) Of course Nathan did not travel **exactly** as suggested by the graph. Suggest some reasons why a more detailed graph would show more variation.

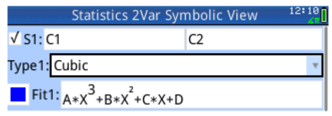
3. Olwyn looked at Nathan's work and suggested he might use a Statistics regression to get a smooth continuous function.

Model the data with an equation

- Press Apps
- Tap Statistics 2Var
- Enter the data as shown



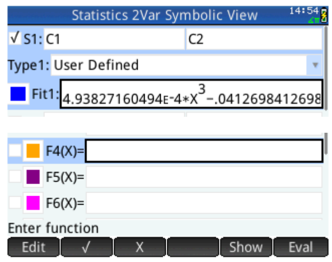
- Press Symb and toggle to cubic
- Press Plot to see scatterplot and tap Fit• to draw the best fit
- Press Symb to view the equation of the graph



Copy the function

- Highlight the function showing in Fit1
- Press Shift View to copy the function
- Press ALPHA and choose Function
- Press Shift Menu to paste to an appropriate function

 Choose function and tap OK



Evaluate

- Press Home to go to the Home screen
- Enter the function name and *x*-value (minutes after 5 o'clock)
- Press Enter

a) What is Olwyn's equation?

b) According to Olwyn's function how far has Nathan travelled at:

i) 5:06

ii) 5:40

iii) 5:50

iv) 6:15

c) Calculate Nathan's average speed (according to Olwyn's function) between:

i) 5:06 and 5:40

ii) 5:42 and 5:55

iii) 5:23 and 5:33

Learning notes

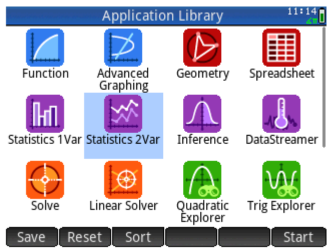
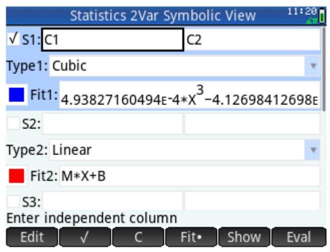
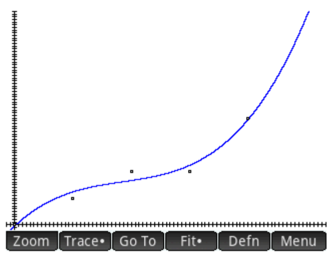
What might Nathan be doing between 5:30 and 5:45?

Q3 Scroll back up the Main window to your function definition. Change the definition to the quartic and the remaining calculations should then follow in the same way.

Equation between two points:

$\frac{y-y_A}{x-x_A}=\frac{y-y_B}{x-x_B}$ This form of the equation of a straight line is an expression of the fact: the gradient between any two points on a straight line is the same.

Statistics regression

To enter data into statistics app • Press Apps Info and tap Statistics 2 Var • Press Symb Setup and set up C1 and C2 as the data being used and the type of graph you wish to draw for the best fit	
Use a cubic regression • Press Apps Info and select cubic • Press Plot Setup to show scatterplot • Press Fit• to draw the line of best fit	

Activity 19 Speed at an instant

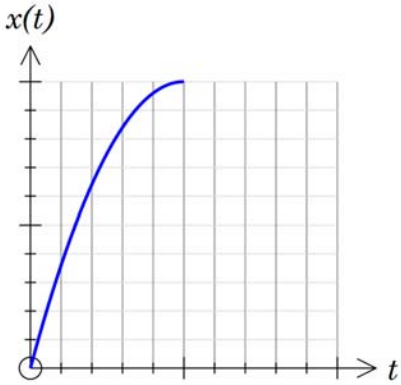
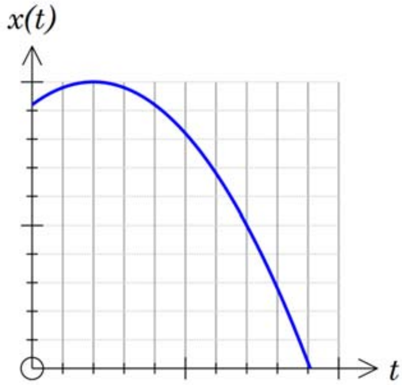
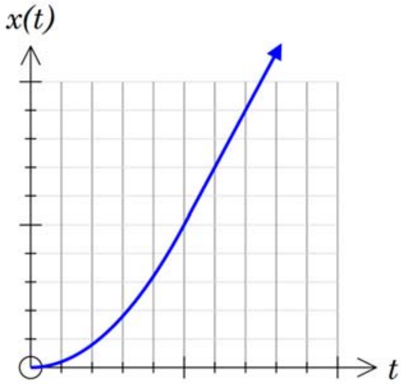
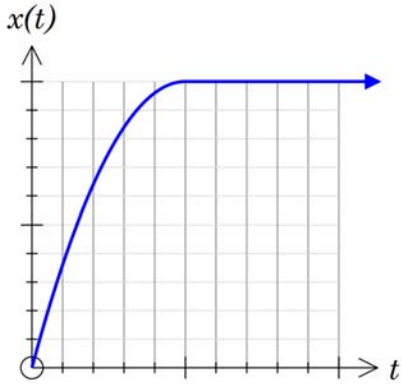
Aim: Develop the concept of speed at an instant.

In the last activity you calculated average speed using two points on a distance-time graph. When the graph is curved, the speed would be continually changing. How can you estimate the instantaneous speed?

1. Match a graph and an equation to scenarios A to D by completing the table.

Scenario	Graph	Equation
A		
B		
C		
D		

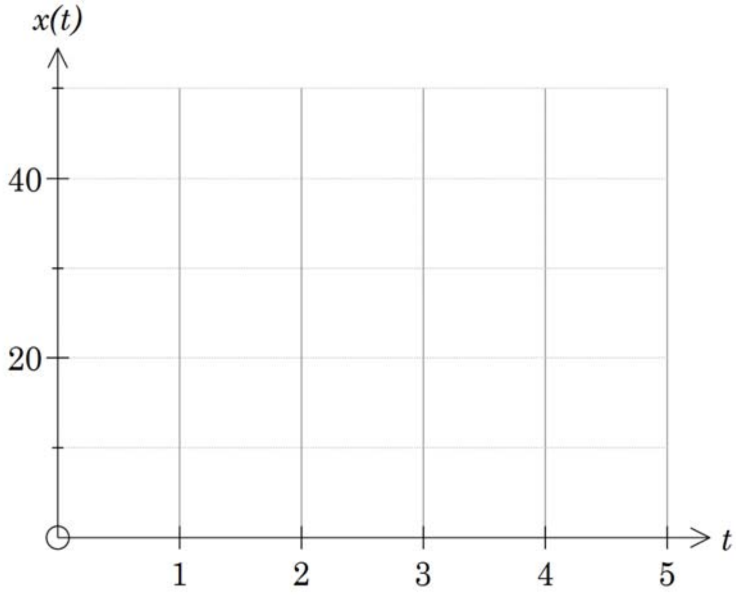
Scenario A	**Scenario B**
A dive from the 10 m high diving board. x is the distance from the water.	Car takes off from a traffic light, accelerates to the speed limit and then travels at the speed limit. x is the distance from the traffic light.
Scenario C	**Scenario D**
Car is travelling at constant speed then brakes suddenly and comes to a stop. x is the distance travelled.	A water powered rocket is launched. x is the height above the ground between launching and maximum height .

Graph 1	Graph 2
	
Graph 3	**Graph 4**
	

Equation (i)	Equation (ii)
$x(t)=\begin{cases}12t^2 & t<5\\ 300 & t\geq 5\end{cases}$	$x(t)=\begin{cases}12t^2 & t<5\\ 60t & t\geq 5\end{cases}$
Equation (iii)	**Equation (iv)**
$x(t)=10-4.9t^2+3t$	$x(t)=30t-3t^2$

2. A skier sliding down a gradient to a ski jump. Consider that part of the slide prior to the gradient changing. The function $x(t) = 1.6t^2$ describes distance travelled in metres versus time in seconds.

a) Graph the distance as a function of time.



b) Complete the table of average speeds.

Interval	Position (start of interval)	Position (end of interval)	Distance travelled in the time interval	Average speed
0 – 1	0	1.6	1.6	
0 – 3				
2 – 3			8	
2.5 – 3				
2.9 – 3				
3 – 3.1				

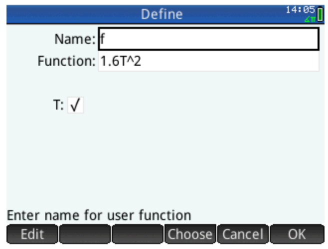
c) On your graph draw a line through the points when $t = 2$ and $t = 3$. Explain why the gradient of this line is the same as the average speed over this interval.

d) At the instant $t = 3$, estimate the speed of the skier.

3. Investigate limits; what happens to the average speed as the time interval decreases.

Define the function

- Press [Shift] [$xt\theta n$ Define]
- Name the function f
- Enter 1.6T^2 for the function
- Tap [OK]



Store and calculate values

Store the time

- Press [CAS Settings]
- Enter a:=3

Decide on the run and store

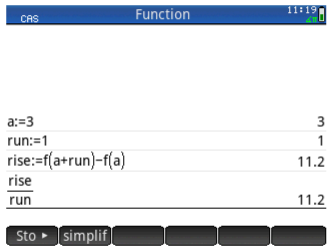
- Enter run:=1

Calculate rise

- Enter rise:=f(a+run)–f(a)

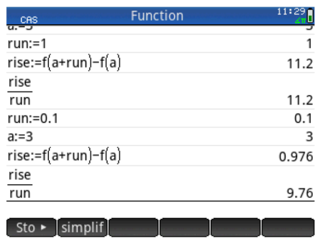
Calculate gradient

- Enter rise/run



Edit the run

- Tap on the line run:=1, press [Enter], edit the value and press [Enter]
- Recalculate the other steps
  Tap on the line rise:=f(a+run)–f(a) press [Enter]
- Tap on the line $\dfrac{\text{rise}}{\text{run}}$ and press [Enter]



a) Complete the table.

Run	Rise	Gradient
1	11.2	11.2
0.5		
0.1	0.976	9.76
0.05		
0.01		
0.0001		

b) Describe what is happening to the gradient as the run gets smaller.

c) Estimate the speed of the skier at the instants

i) $t = 3$

ii) $t = 4$

iii) $t = 2.5$

Learning notes

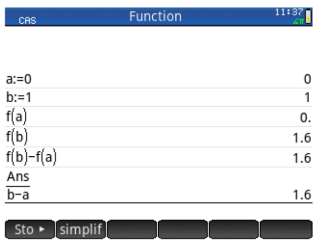
Q1 You can plot the equations in Function. Use pinch and zoom to adjust the window to match the shape shown in the graphs.

This question is a good opportunity to discuss features of quadratic graphs and how they link with the equations. For example two graphs have a minimum turning point and two a maximum turning point. How does this help in identifying which equation matches (or can't possibly match) which graph?

Q2 There are numerous ways Prime could be used to do this, e.g.

In CAS

- Define the function
- Store values for start time and end time
- Calculate positions at these times
- Calculate the distance travelled
- Calculate the average speed
- Change the values for start and end times and press Enter with the cursor in start time.



Q3 b) Edit the value of a.

Activity 20 Gradient functions

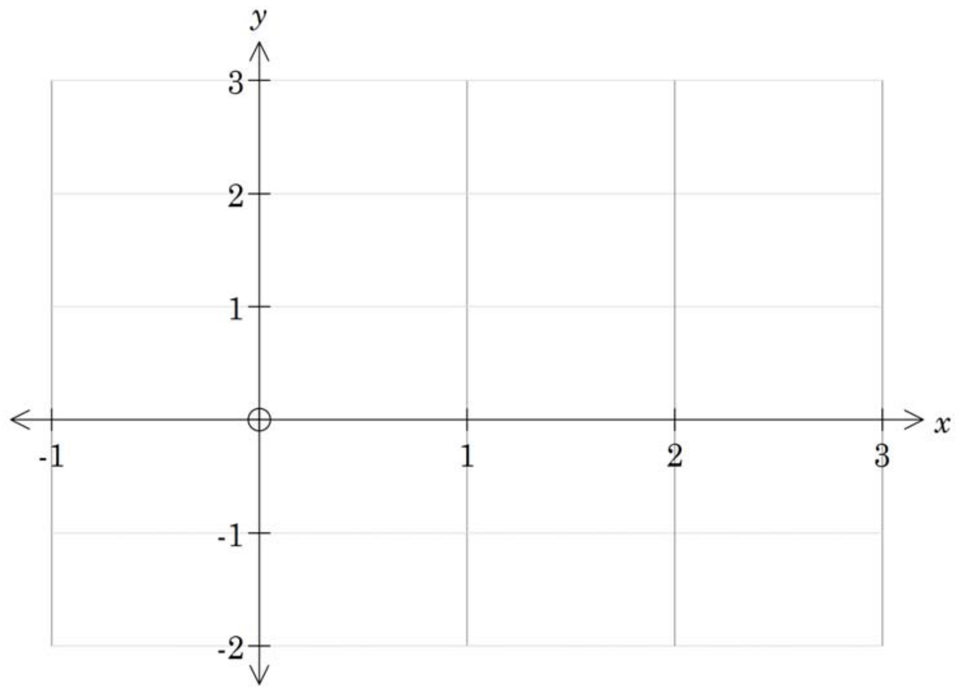
Aim: Sketch curves and relate key features of a function with its derivative function.

1. Consider the graph of $y = f(x)$ where $f(x) = 2x^3 - 5x^2 + 3x$.
   (See Learning notes for instructions).

   a) Determine the coordinates of the x-intercepts.

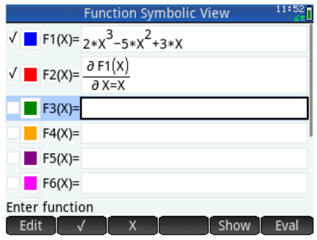
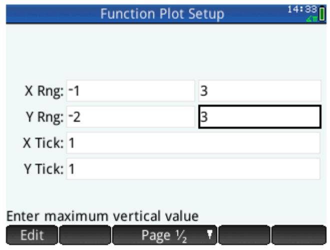
   b) Determine the coordinates of the turning points.

   c) Sketch the graph on the grid below.
      First plot the x-intercepts and stationary points. Then draw in the curve.



Note: Plotting key features first is useful for transcribing graphs to paper.

2. Graph the derivative function, $y = f'(x)$.

<table>
<tr>
<td>Enter the derivative as a function.
Press A and select the Function app
Enter F1(X)
Tap in F2(X)
Press Tap the derivative function
Complete the entry as shown</td>
<td></td>
</tr>
<tr>
<td>Draw graphs
Press Plot Setup to graph
Press Shift Plot Setup to set view window to match the grid above</td>
<td></td>
</tr>
</table>

a) Determine the coordinates of the x-intercepts.

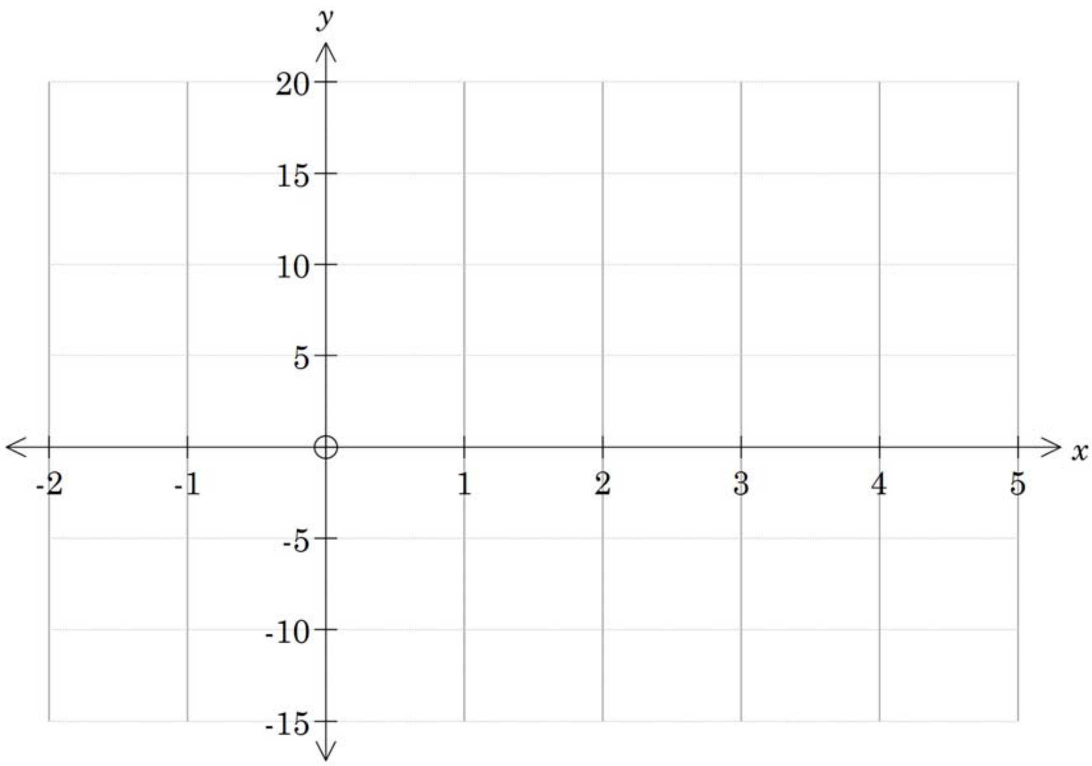
b) Determine the coordinates of any stationary points.

c) Sketch the graph on the same axes as the function was plotted. Use a different colour.

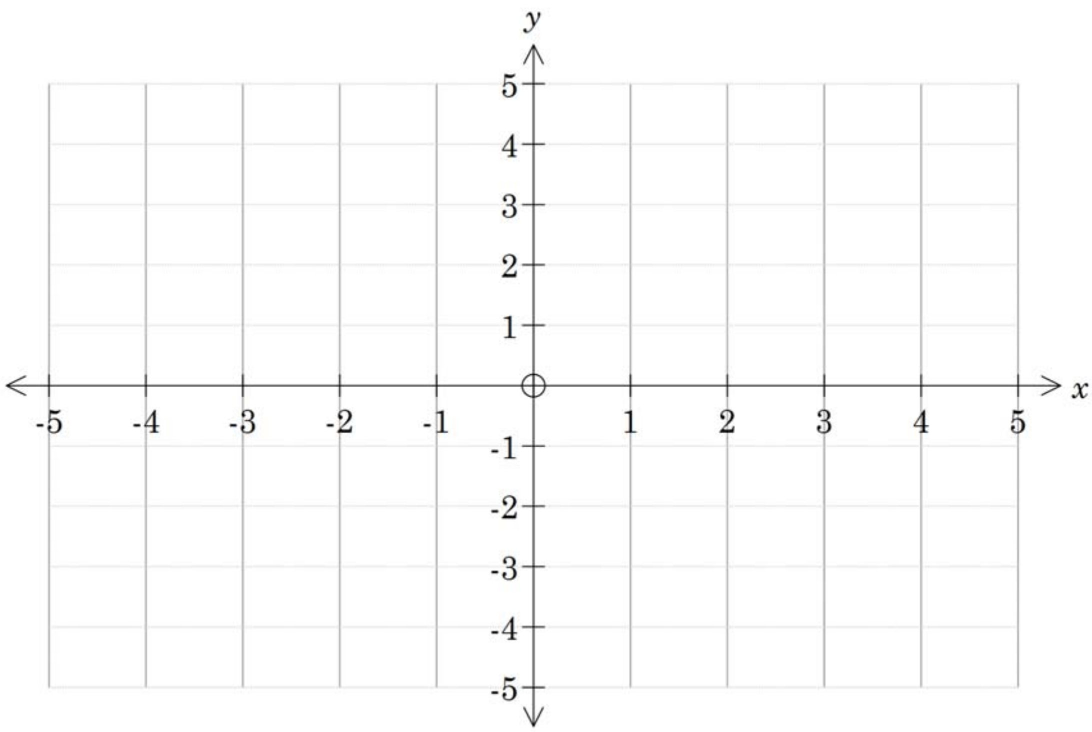
3. Describe all the connections you can identify between the key features of the graph of the function (Q1) and its derivative (Q2).

4. Draw the graph of each function and the graph of its derivative on the grid provided. Calculate, plot and label key features of each graph.

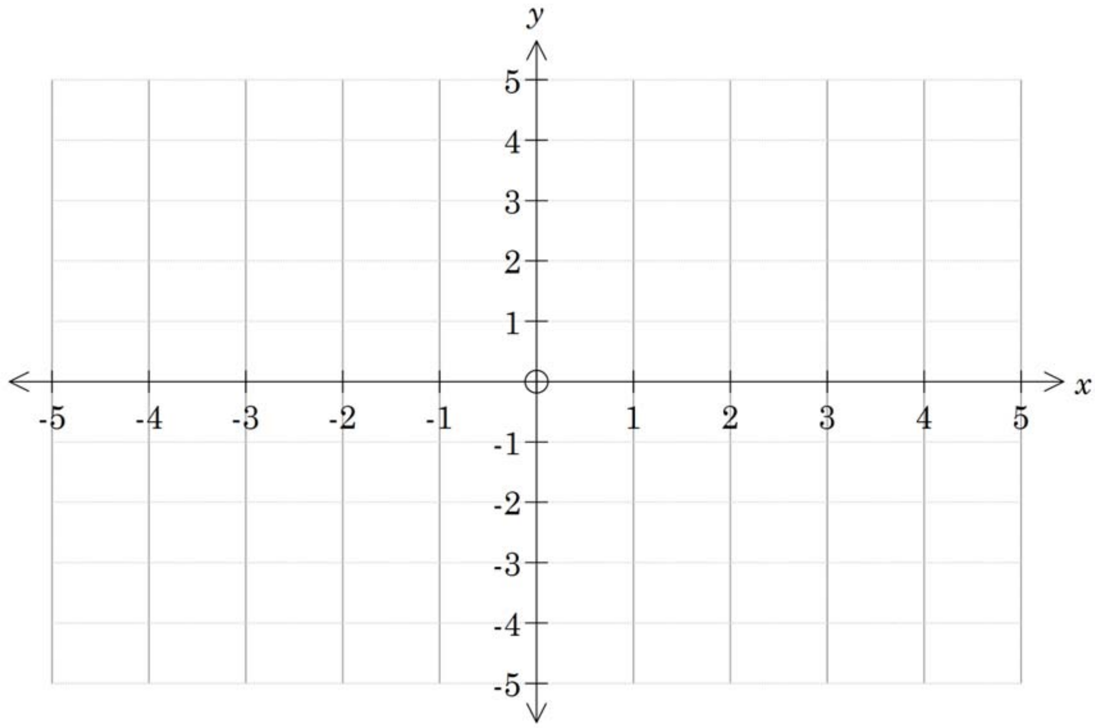
a) $y = x^4 - 6x^3 + 9x^2 - x - 6$



b) $y = x + \frac{1}{x}$



c) $y = \dfrac{x^3 - 9}{x^2 + 2}$



5. Use your work in this investigation to complete the table.

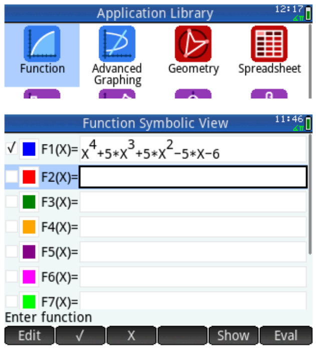
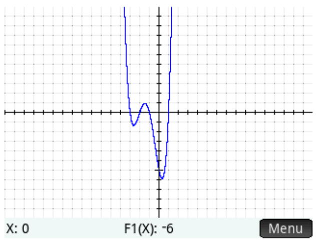
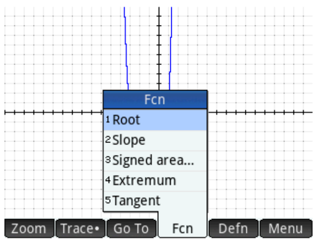
Feature of function	Corresponding feature(s) of graph of derivative function
x-intercept	none
Local maximum	
Local minimum	
Turning point	
	Turning point

Learning Notes

The main part of this activity is making connections between graphs of a function and its derivative or gradient function. It will be helpful to keep in mind that the gradient at a stationary point is 0.

When sketching a graph that you have displayed using technology:

- Ensure the window is appropriate, i.e. match the calculator window to the grid provided or adjust the scale to show the features you want;
- Calculate values for the key features;
- Plot the key features;
- Sketch the graph.

Draw the graph • Press Apps Info • Tap Function • Enter the function as F1(X) • Press Enter	
Set the window • Press Symb Setup and Shift Plot Setup • set X Rng: and Y Rng to match the given axes **Roots** • Tap Fcn 1.Root • Move the cursor closer to next root and repeat **Locate turning points** • Tap Fcn 4.Extremum • Move the cursor closer to next turning point and repeat	

Key features of graphs will vary, depending upon the function. You may wish to include:

- intercepts;
- stationary points (local maxima, minima and stationary points of inflection);
- asymptotes; and
- behaviour as $x \to \pm\infty$.

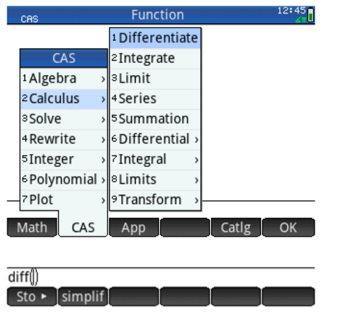
Activity 21 Differentiate

Aim: Calculate derivatives.

1. Complete the table.

Calculate a derivative using Prime

- Press CAS Settings to open a CAS window
- Press Mem B
- Select [CAS > Calculus > Differentiate
- Enter desired expression.

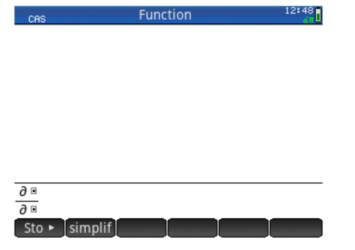


Expression	Formal mathematical expression	Prime output
diff$(x\,^\wedge 3)$	$\dfrac{d}{dx}(x^3)$	
diff$(a \times x\,^\wedge 3 + b \times x + 3)$		
diff$((x\,^\wedge 3)\,^\wedge(1/2))$		
diff$(x\,^\wedge 1.5)$		
diff$(a \times x\,^\wedge n)$		
diff$(x\,^\wedge 3 - 7.5x\,^\wedge 2 + x)$		
diff$(t\,^\wedge 3 - 7.5t\,^\wedge 2 + t)$		
diff$(t\,^\wedge 3 - 7.5t\,^\wedge 2 + t, t)$		
diff$(x\,^\wedge 3 - 7.5x\,^\wedge 2 + x) \mid x = 3$		
diff(diff$(x\,^\wedge 3 - 7.5x\,^\wedge 2 + x)$)		
diff$(x\,^\wedge 3 - 7.5x\,^\wedge 2 + x, x, 2)$		
diff$(x\,^\wedge 3 - 7.5x\,^\wedge 2 + x, x, 3)$		

2. An alternative to the diff command is to use a template.

Calculate derivatives using template

- Press
- Tap the derivative function
- Enter the variable and expression in the appropriate part of the template
- Press Enter



Enter each expression as shown and record the output, simplifying where appropriate.

a) $\dfrac{d}{dx}\left(4x^2-5x\right)$

b) $\dfrac{d}{dx}\left(\left(4x^2-5x\right)\left(6x^4-3x^3+2x\right)\right)$

c) $\dfrac{d}{dx}\left(5x^7-\dfrac{31}{x^2}\right)$

d) $\dfrac{d}{dx}\left(\dfrac{x^2}{x^4+7.5x^2-5}\right)$

e) $\dfrac{d}{dx}\left(x^2-\sqrt[4]{x^3}-7x\right)$

f) $\dfrac{d}{dt}\left(5t-\dfrac{3}{\sqrt{t}}\right)$

Learning Notes

Make sure you agree with or understand the Prime output. You should also be able to calculate all the derivatives in this activity without the aid of technology.

Activity 22 Modelling motion

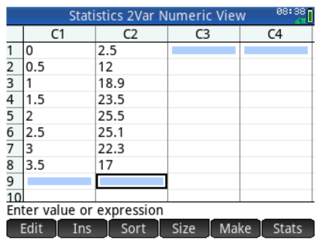
Aim:	Determine and use time-related derivatives for motion in a straight line; velocity, speed and acceleration. Model motion along a straight line.

Mitch throws a cricket ball straight up in the air. Peter records the throw on his iPad and gets the following data on the height of the ball.

Time (seconds)	0	0.5	1.0	1.5	2	2.5	3	3.5
Height (metres)	2.5	12	18.9	23.5	25.5	25.1	22.3	17

Model this data to obtain a height function

- Enter the data into Statistics 2VAR App
- Draw a scatter graph
- Use the regression that fits the shape of your graph



1. Record the height function.

2. Use your model to determine:

   a) the velocity function

   b) when the velocity is 0

   c) the acceleration function

   d) the maximum velocity in the interval $0 \le t \le 4.4$

   e) the maximum speed in the interval $0 \le t \le 4.4$

   f) the maximum height

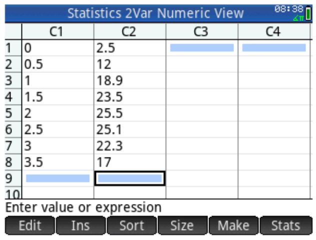
Learning notes

To model data:

- Enter the data into Statistics
- Draw the graph
- Choose a regression that fits the shape of the data

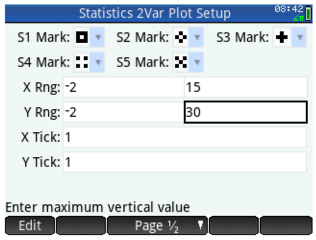
Enter the data into Statistics

- Open Statistics 2VAR
- Select [Edit | Clear All]
- Enter the data, time in C1 and height in C2



Draw a scatter graph

- Press Plot Setup
- To check scale of graph press Shift Plot Setup and set x and y range
- Press Plot Setup

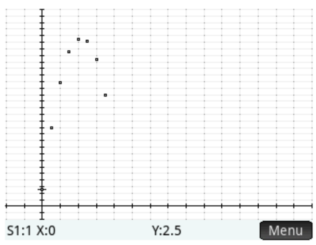


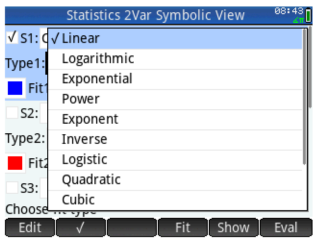
Look at the graph and see what the shape looks like. Does it look like a straight line, a parabola, … ?

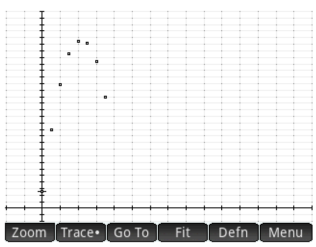
Do regression calculation

- Press Symb Setup and select the regression line you have decided on. The equation will be below this.
- Press Plot Setup and Tap Menu then Fit•

The graph is drawn. If it fits your points well then it would seem to be an appropriate model.



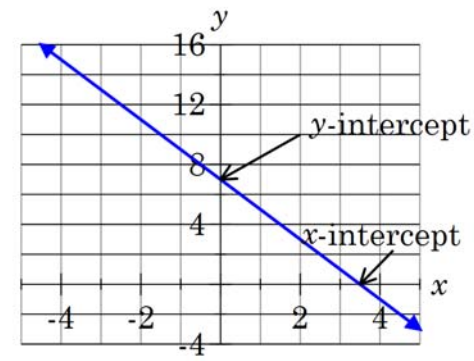




Solutions

Activity 1 Features of graphs

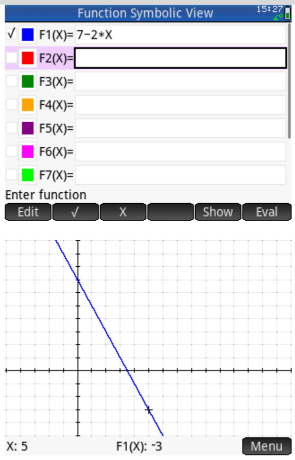
1. $y = 7 - 2x$



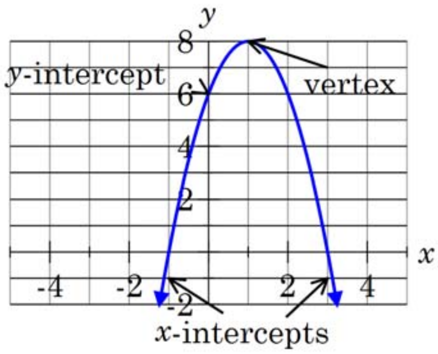
a) (0, 7)

b) (3.5, 0)

c) –2



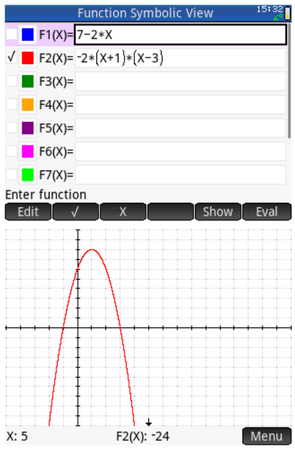
2. $y = -2(x+1)(x-3)$



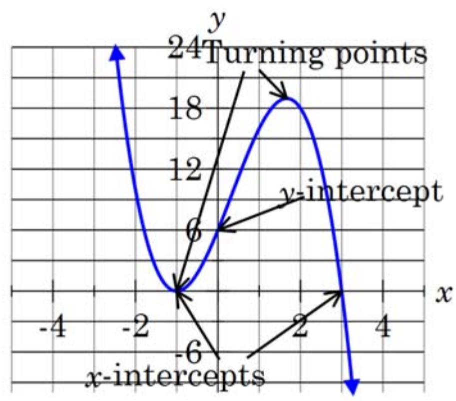
a) (0, 6)

b) (–1, 0) and (3, 0)

c) (1, 8)



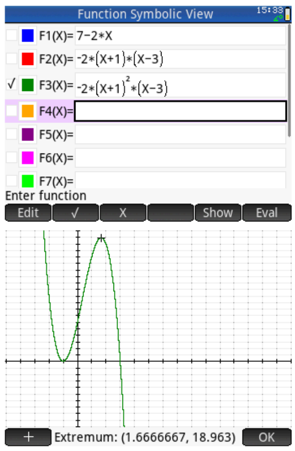
3. $y = -2(x+1)^2(x-3)$



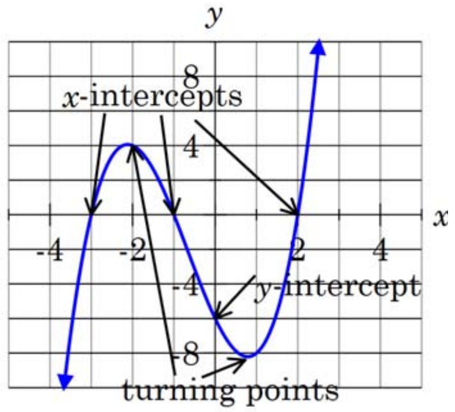
a) (0, 6)

b) (–1, 0), (3, 0)

c) (–1, 0), (1.667, 18.96)



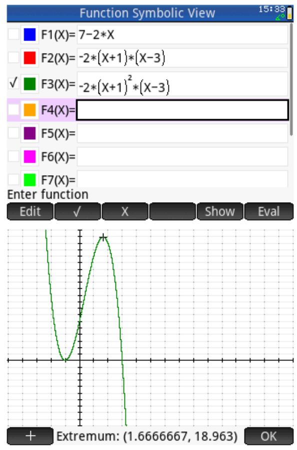
4. $y=(x-2)(x+1)(x+3)$



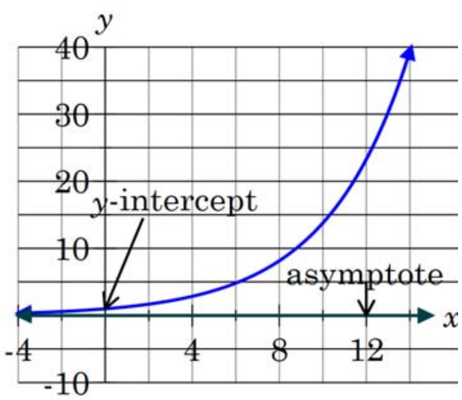
a) (0, –6)

b) (–3, 0), (–1, 0) and (2, 0)

c) (0.786, –8.21) and (–2.12, 4.06)



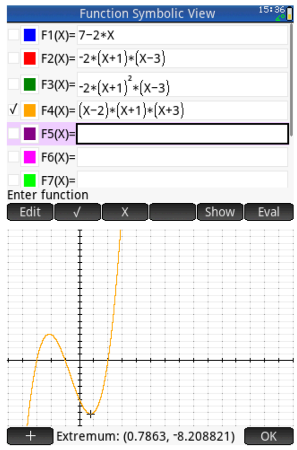
5. $y=1.3^x$



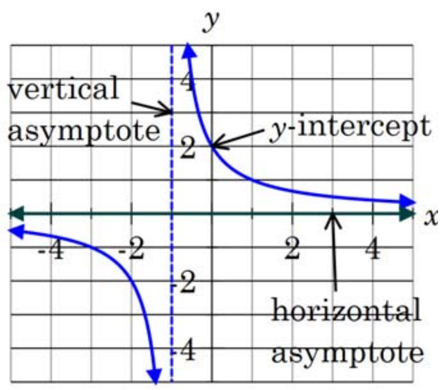
a) (0, 1)

b) none

c) $y=0$



6. $y=\dfrac{2}{x+1}$

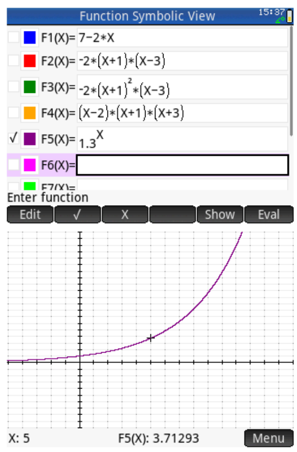


a) (0, 2)

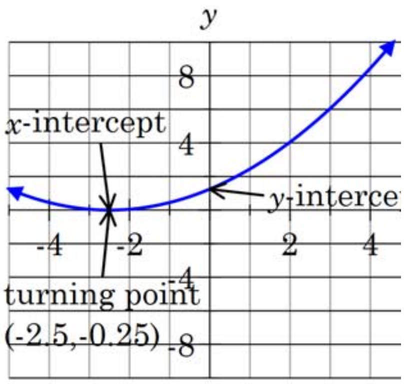
b) none

c) $y=0$

d) $x=-1$



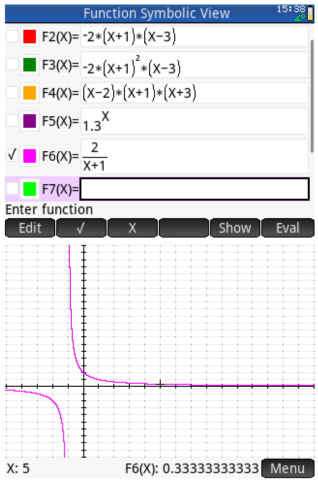
7. $y = 0.2(x + 2.5)^2$



a) (0, 1.25)

b) (–2.5, 0)

c) Minimum at (–2.5, 0)



Activity 2 How big is the package

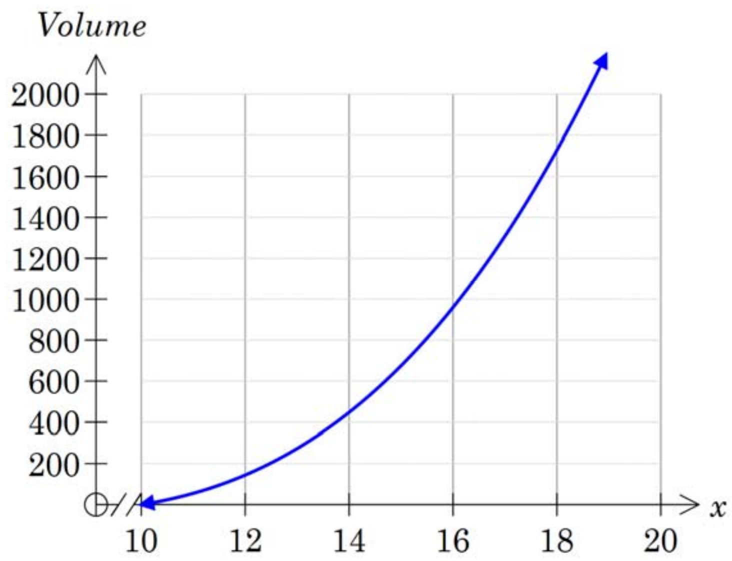
1.

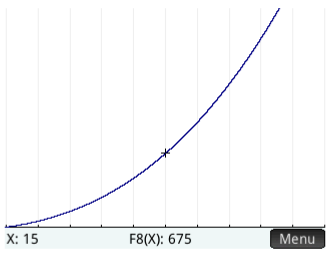
Box	Length (cm)	Width (cm)	Depth (cm)	Volume (cm^3)
A	25	19	15	7125
B	22	16	12	4224
C	15	9	5	675
D	12	6	2	144

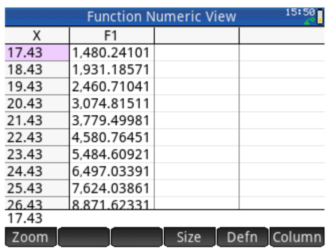
2. If the box is length x, then
the width is $x - 6$, as it is 6 cm less and
the depth is $(x - 6) - 4 = x - 10$, as it is 4 cm less than the width.

The volume is length × width × depth i.e. $V = x(x-6)(x-10)$.

3. The depth is bigger than 0 so the length must be at least 10 cm i.e. $x > 10$







4.

a) 1480 cm^3 (3 s.f.)

b) 36 300 cm^3

c) 19.5 cm

d) At least 19.92 × 13.93 × 9.92

Activity 3 Circles

1. $x^2 + y^2 = 1$ Pythagorean theorem.

2. For a circle centred at the origin with radius r units: $x^2 + y^2 = r^2$.

3.

Equation	Centre	Radius
$(x-1)^2 + y^2 = 1$	(1,0)	1
$(x-2)^2 + (y-1)^2 = 1$	(2,1)	1
$(x+1)^2 + (y+3)^2 = 4$	(–1,–3)	2
$(x-A)^2 + (y-B)^2 = R^2$	(A,B)	R

4. Completing the square,

$$x^2 - 6x + y^2 = -8$$
$$(x-3)^2 - 9 + y^2 = -8$$
$$(x-3)^2 + y^2 = 1$$

5. $(x+2)^2 + (y-3)^2 = 16$

6. Centre (2.5, –4), radius 6

Activity 4 Phone costs

1. a) 9
   b) 85
   c) \$79.16
   d) \$170.84
2.

	calls	minutes	Credit remaining
$c(10,250)$	10	250	\$23.60
$c(50,150)$	50	150	\$97
$c(72,175)$	72	175	\$66.17
$c(32,220)$	32	220	\$41.72
$c(40,200)$	40	200	\$56.40

3. 70
4. a) \$8.32
   b) \$45.26
   c) 202
5. a) $-0.89m + 246.1$
   b) $-0.89\,\text{mins} + 246.1$
   c) $-0.39x - 0.89y + 250$
   d) $-1.78m + 246.1$
   e) $-0.39x - 1.78y + 250$
6. a) $c(n,m,t,d) = 250 - 0.39n - 0.89m - 0.29t - 2d$
   b)

	calls	minutes	SMS	Data Mb)	Credit (\$)
$c(10,150,75,0)$	10	150	75	0	\$90.85
$c(10,90,350,3)$	10	90	350	3	\$58.50
$c(72,175,21,4)$	72	175	21	4	\$52.08
$c(32,100,60,12)$	32	100	60	12	\$107.12
$c(21,199,73,0)$	21	199	73	0	\$43.53

7.
   a) 7
   b) 19
   c) 1
   d) 1
   e) 12
   f) 2
8. The function $c(n,m) = 250 - 0.39n - 0.89 \times \text{CEILING}(m)$ would enable m to be entered as a decimal rather than being rounded up first.

Activity 5 Trigonometric graph transformations

1. $y = \sin x$:

   x-intercepts at multiples of 180°

   y-intercept at the origin

   period 360°

   amplitude 1 unit

2. $y = a\sin x$

   Vertical dilation by factor a.

3. $y = \sin x + v$

   Vertical translation v units.

4. $y = \sin(bx)$

   Horizontal dilation factor $\frac{1}{b}$.

5. $y = \sin(x + h)$

   Horizontal translation $-h$ units.

6. Note: Other answers are possible.

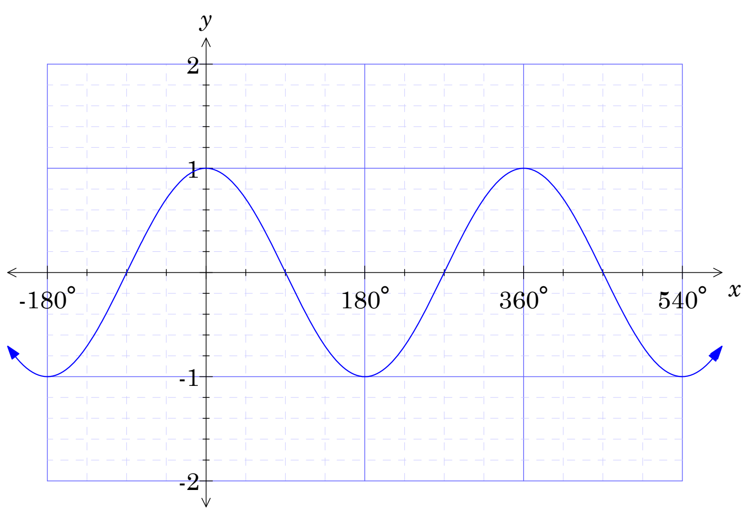
   a) $y = 2\sin(3x)$

   b) $y = 3\sin(x - 30°)$

   c) $y = \sin(2x - 1)$

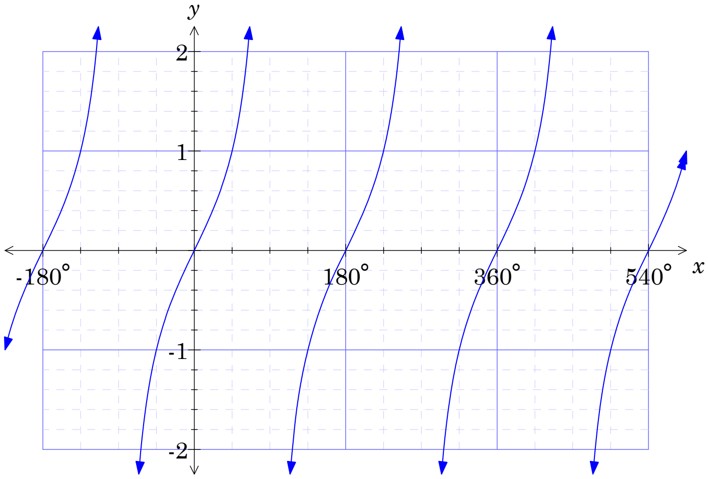
   d) $y = -\sin\left(\frac{x}{2}\right) + 1$

7. $y = \cos x$

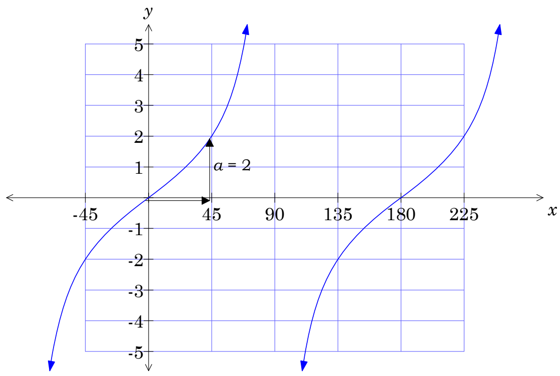


8. Transformations for $y = a\cos(b(x+h)) + v$ are the same as those for the sine function above.

9. $y = \tan x$



10. Transformations are the same as those for sine and cosine. Note that the a value can be determined by looking at the vertical movement required to move to the right from a point of inflection to a point halfway to the asymptote. For example, the graph shows $y = 2\tan x$



11. a) $y = \tan(x + 30°)$

b) $y = \tan(3x) - 2$

12. Transformations to all functions in radians are the same as those for degrees. Care must be taken with horizontal dilations. In general, b represents the number of cycles in $360°$ i.e. 2π radians.

13. As for sine

14. Same as in degrees. b is the number of cycles in $180°$ i.e. π radians .

15. a) $y = 3\cos\left(2\left(x - \frac{\pi}{6}\right)\right)$

b) $y = -4\sin(3x)$

c) $y = 0.5\tan(3x)$

d) $y = 0.8\cos\left(\frac{\pi x}{4}\right)$

Activity 6 Modelling with trigonometric functions

1.

a) The height of a point moving around a ferris wheel varies periodically, rising and falling in a regular cycle.

$d = 6.0\sin(0.21t - 1.6) + 7.0$

b)

i) Radius 6.0 metres

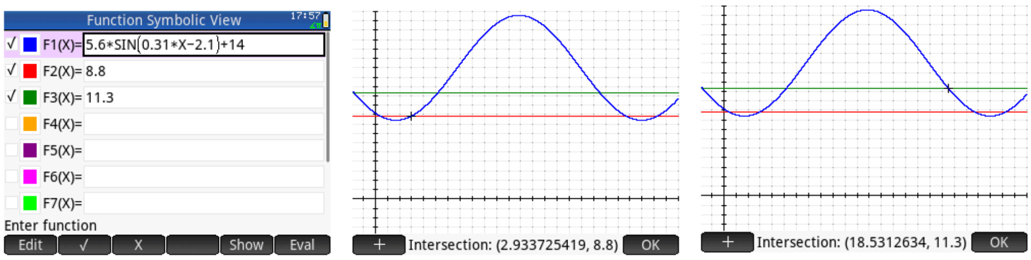
ii) Minimum height 1 m, maximum height 13 m

iii) Period $\dfrac{2\pi}{0.21} \approx 30$ s

2. $d = -6.0\cos(0.21t) + 7.0$

3. $d = -6.0\sin(0.21t) + 7.0$

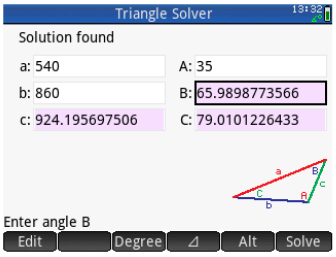
4. $d = 5.6\sin(0.31t - 2.1) + 14$



Ship can enter port 3.4 hours after midnight and exit 17.9 hours (rounded down) after midnight. The corresponding times are approximately 3:25 a.m. and 5:55 p.m. respectively (nearest 5 minutes).

Activity 7 Window dressing

1.



a) 81.5°

b) 60.9°

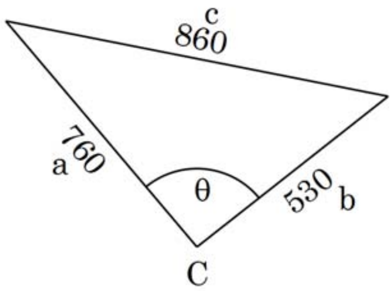
c) 754 mm

d) 3188 cm²

e) $62.64

2.

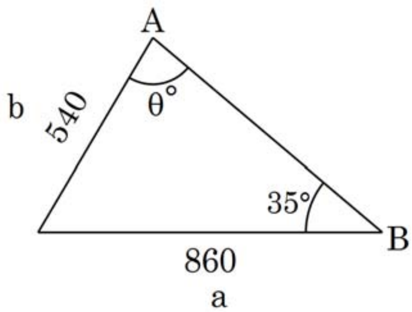
$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$86^2 = 76^2 + 53^2 - 2 \times 76 \times 53 \cos\theta$$
$$\theta = 81.5°$$



3.

a)

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin\theta}{860} = \frac{\sin 35°}{540}$$



b) $\theta \approx 114°,\ 66°$

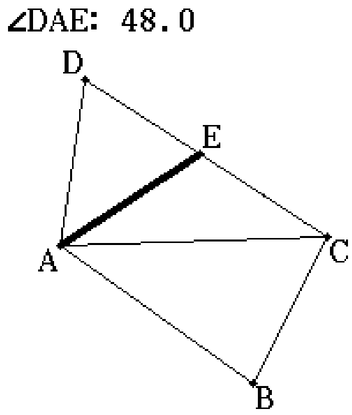
Angles are supplementary.

c) The frame contained an obtuse angle BCD but the glazier cut the glass with an acute angle.

4.

a) Angle DAC ≈ 79°, 31°

b) The glazier needs to remove an isosceles triangle, ADE in the diagram at right, with AD = AE = 540 mm.



5.

c) 49.4°, 130.6°

d) Triangle ACD becomes isosceles and only one triangle is possible.

e)

i) 493.3 mm. At this length, angle ADC is 90° and triangle ACD is unique.

ii) This is the minimum distance from point A to the ray CE. A smaller length will not intersect the ray and hence triangle ACD will not exist.

Activity 8 Pascal's triangle

1.

Row #		Row sum
	1	1
1	1	2
2	1 2 1	4
3	1 3 3 1	8
4	1 4 6 4 1	16
5	1 5 10 10 5 1	32
6	1 6 15 20 15 6 1	64
7	1 7 21 35 35 21 7 1	128

2.

3.

a) 45

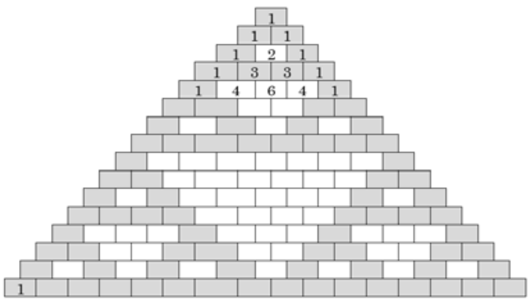
b) 45

c) 455

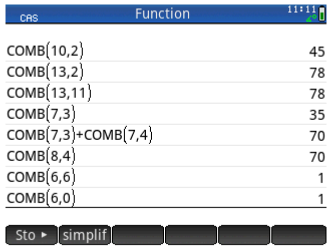
d) 3rd or 11th number in the 13th row or
2nd or 77th number in 78th row.

4. The sum is double the sum in the previous row.
Each element is added to two numbers beneath apart from the ends. These are effectively doubled by adding the extra ones at the beginning and end of the row.

5.



Activity 9 Combinations and Pascal's triangle

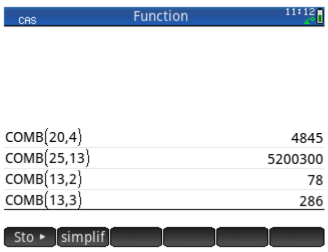


1. a) i) 6 ii) 21
   b) 15 people and 105 handshakes
2. a) 45
   b) 78
   c) 78
   d) 35
   e) 70
   f) 70
   g) 1
   h) 1
3. a)

$\binom{4}{0}=1$	$\binom{4}{1}=4$	$\binom{4}{2}=6$	$\binom{4}{3}=4$	$\binom{4}{4}=1$
$\binom{5}{0}=1$	$\binom{5}{1}=5$	$\binom{5}{2}=10$	$\binom{5}{3}=10$	$\binom{5}{4}=5$
$\binom{6}{1}=6$	$\binom{6}{2}=15$	$\binom{6}{3}=20$	$\binom{6}{4}=15$	$\binom{6}{5}=6$
$\binom{7}{1}=7$	$\binom{7}{2}=21$	$\binom{7}{3}=35$	$\binom{7}{4}=35$	$\binom{7}{5}=21$

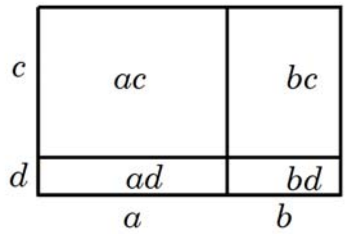
   b) These are the same numbers as Pascal's triangle with the "choose from" equalling the row number and the "number chosen" being one less than the position in the row. I.e. n choose r is the r+1th element in the nth row.
4. a)
   i) 1
   ii) 21

   b)
   i) 7
   ii) $\binom{7}{2}$
5. a) $\binom{20}{4}=4845$
   b) $\binom{25}{13}=5\ 200\ 300$
   c) The fourth element is $\binom{13}{3}=286$



Activity 10 Binomial expansion

1. Area of large rectangle is $(a+b)(c+d)$.
   This is the same as the sum of the areas of the four smaller rectangles area i.e. $ac+ad+bc+bd$



2.
   a)
      i) $ax+ay+bx+by+cx+cy$ 6 terms
      ii) $ax+ay+bx+by+cx+cy+dx+d$ 8 terms
      iii) $ax+ay+az+bx+by+bz+cx+cy+cz$ 9 terms
      iv) $ax+ay+az+bx+by+bz+cx+cy+cz+dx+dy+dz+ex+ey+ez$ 15 terms
   b) mn
   c) Drawing a rectangle, divide one side into m pieces and the other side into n pieces. This will divide the large rectangle into mn pieces.
   d)
      i) $a^2+2ab+ac+b^2+bc$
      ii) 5
      iii) There are two like terms, i.e. two rectangles with area ab. That is an initial expansion has 6 terms before collecting like terms.

3.
   a)
      i) 8
      ii) 12
      iii) 18
      iv) 16
      v) 16
   b) The product of the number of terms in each bracket.
   c) Already established for 2 brackets in Q3
      For three brackets we can imagine the product as a 3D rectangular prism which gets split up into smaller pieces.

4.

a)

Expression	Expansion
$(a+b)^2$	$a^2+2ab+b^2$
$(a+b)^3$	$a^3+3a^2b+3ab^3+b^3$
$(a+b)^4$	$a^4+3a^3b+6a^2b^2+3ab^3+b^4$
$(a+b)^5$	$a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5$
$(a+b)^6$	$a^6+6a^5b+15a^4b^2+20a^3b^3+15a^2b^4+6ab^5+b^6$

b)

$(a+b)^2$	1 2 1
$(a+b)^3$	1 3 3 1
$(a+b)^4$	1 4 6 4 1
$(a+b)^5$	1 5 10 10 5 1
$(a+b)^6$	1 6 15 20 15 6 1

c) The coefficients are the elements of Pascal's triangle.
The power is the row number.

5.

$$(2x^2-5)^3=(2x^2)^3+3(2x^2)^2(-5)+3(2x^2)(-5)^2+(-5)^3$$
$$=8x^6-60x^4+150x^2-125$$

i.e consider $2x^2$ as one term and (-5) as the other.

Activity 11 Exponential Functions

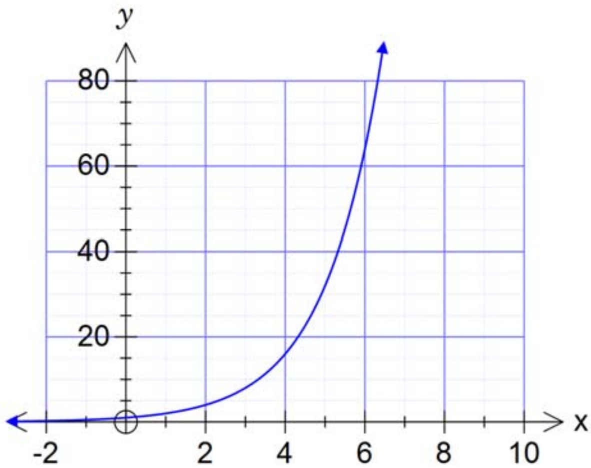
1.

a)

x	0	2	4	10
$y=2^x$	1	4	16	1024

x	-2	-1	1.2	1.5
$y=2^x$	$\frac{1}{4}$	$\frac{1}{2}$	2.30	2.83

b)



c)

As $x \to \infty,\ y \to \infty$

As $x \to -\infty,\ y \to 0^+$

2.

Equation	Key features	Graph
A	5	III
B	4	VI
C	1	II
D	6	I
E	2	IV
F	3	V

Equation of asymptote
$y=8$
$y=-4$
$y=0$
$y=-1$
$y=0$
$y=0$

3.

a) As $x \to \infty, y \to \infty$

b) As $x \to -\infty, y \to c$

c) $y = c$

d) $2^{-b} + c$.

e) $c < 0$.

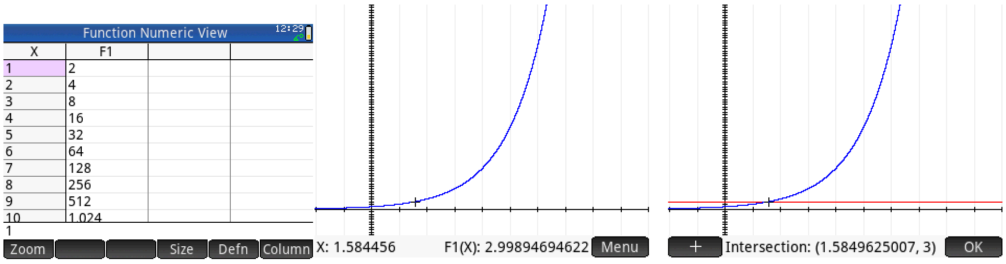
Activity 12   Solve exponential equations

1.

a) Between 1 and 2

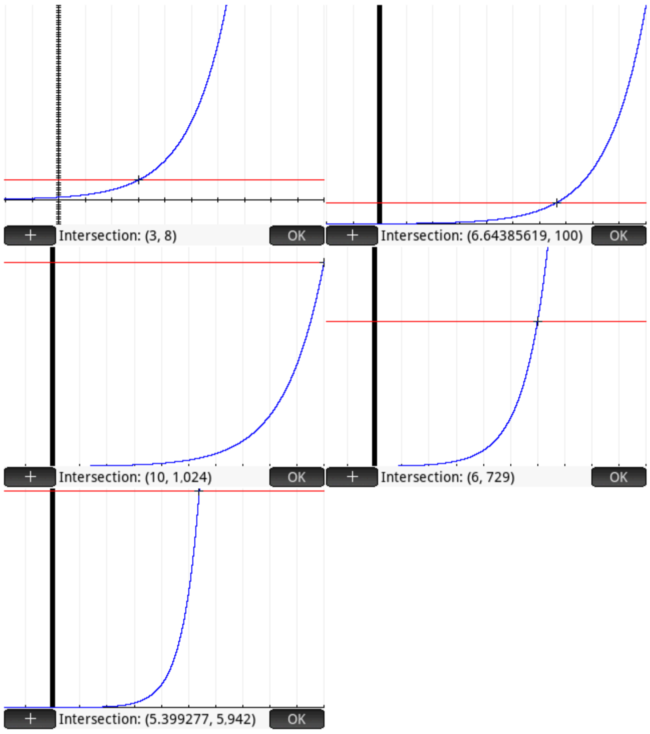
$x = 1.58$ (2 d.p.) , answers may vary with the size of the view window

b) 1.5850



c) 1.5850

2.



a) 3

b) 6.644

c) 10

d) 6

e) 5.399

3.

a) $1–2^n$

b) 2^n

c) 9

d) 8

e) 5.044

f) 5.044

g) 5.044

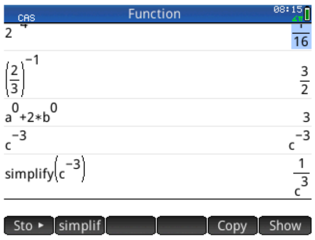
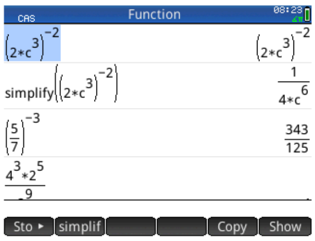
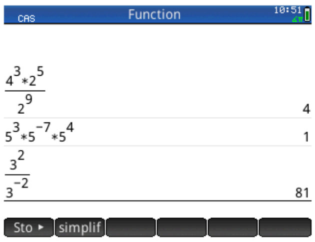
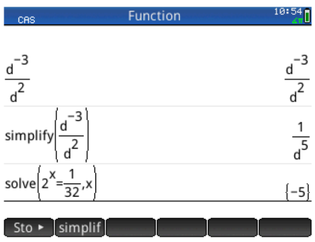
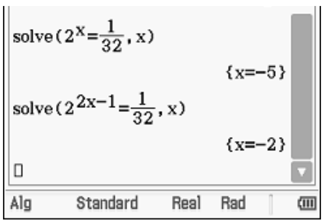
h) 0.75

i) 13

j) –1.527

Activity 13 Index laws

Prime	Rule(s) used by CAS
1. $\frac{1}{16}$	$a^{-n} = \frac{1}{a^n}$
2. $\frac{3}{2}$	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad a^{-n} = \frac{1}{a^n}$
3. 3	$a^0 = 1$
4. $\frac{1}{c^3}$	$a^{-n} = \frac{1}{a^n}$
5. $\frac{1}{4c^6}$	$(ab)^n = a^n b^n \quad a^{-n} = \frac{1}{a^n}$
6. $\frac{343}{125}$	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad a^{-n} = \frac{1}{a^n}$
7. 4	$\frac{4^3 \times 2^5}{2^9} = \frac{\left(2^2\right)^3 2^5}{2^9} = \frac{2^6 2^5}{2^9} = 2^2$
8. 1	$a^n a^m = a^{n+m}$
9. 81	$\frac{a^m}{a^n} = a^{m-n}$
10. $\frac{1}{d^5}$	$\frac{a^m}{a^n} = a^{m-n}$
11. -5	$2^x = \frac{1}{2^5} = 2^{-5}$ $x = -5$
12. -2	$2^{2x-1} = \frac{1}{2^5} = 2^{-5}$ $2x - 1 = -5$ $x = -2$

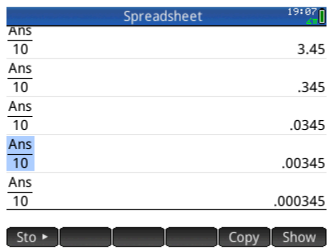
Activity 14 Scientific notation

1.

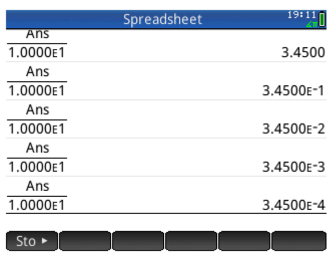
Standard mode	Decimal mode	Decimal number	Sci Not
345000	345000	345 000	3.45×10^{5}
3450000	3450000	3 450 000	3.45×10^{6}
34500000	34500000	34 500 000	3.45×10^{7}
345000000	345000000	345 000 000	3.45×10^{8}
3450000000	3450000000	3 450 000 000	3.45×10^{9}
34500000000	3.45E+10	34 500 000 000	3.45×10^{10}
345000000000	3.45E+11	345 000 000 000	3.45×10^{11}

2. In standard mode all the digits are displayed. In Decimal mode large numbers are displayed in scientific notation.

3.



Prime display	Number	Sci Not
34.5	34.5	3.45×10^{1}
3.45	3.45	3.45×10^{0}
.345	0.345	3.45×10^{-1}
.0345	0.0345	3.45×10^{-2}
.00345	0.00345	3.45×10^{-3}
.000345	0.000345	3.45×10^{-4}
.0000345	0.0000345	3.45×10^{-5}



a) 6.48×10^{-26}

b) 8.44×10^{7}

4.

a) 7.99×10^{7} electrons

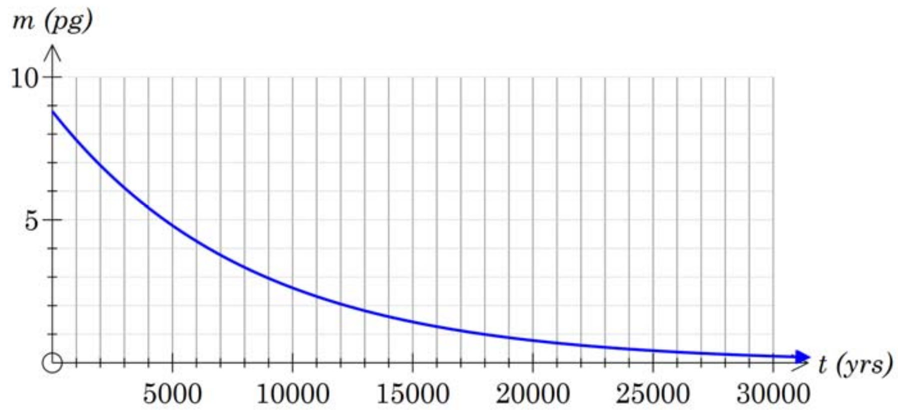
b) 6.0×10^{24} kg.

Activity 15 Carbon dating

1.

a) $W = 8.8 \times 2^{\frac{-t}{5720}}$

b)



c)

i) 8.66 pg

ii) 6.12 pg

iii) 1.43 pg

d)

i) 19 000 years

ii) 57 000 years

2.

a) When fixed in the organism,

$$\frac{C14}{C12} = 10^{-12}$$

$$C14 = C12 \times 10^{-12}$$

b)

Sample	C14 (pg)	Age
Charcoal	2	5720
Tree	0.42	830
Peat	0.063	13200
Bone	7.18×10^{-3}	233
Tooth	0.0061	1760

c) $t = 8250 \ln\left(\frac{W_0}{W_t}\right)$

3.

a) $P(t) = 69 \times 0.9426^t$

b)

Sample	C14 (%)	Years since 1965	Age
Bone	137%	10.6	1975
Tooth	163%	1.5	1966
Bone	129%	14.9	1980

4.

a) 1983

b) 1420 years ago

Extension

The decay of C14 has little effect on the accuracy due to the relatively small number of years involved compared to the half-life of C14.

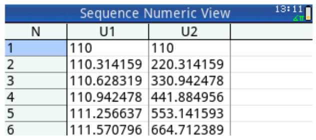
Our model for dating the old objects assumes a constant proportion of C14:C12. It has been shown that small fluctuations exist at various points in the past, which are taken into account to increase the accuracy of the approximation of the age of an object.

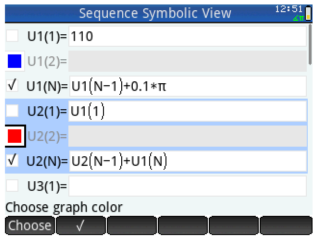
Activity 16 Masking tape

1. a)

Winding	Diameter (mm)	Length of winding	Total length
1	35	110	110
2	35.1	110.3	220.3
3	35.2	110.6	330.9
4	35.3	110.9	441.8
5	35.4	111.3	553.2

b) $L_n = L_{n-1} + 0.1\pi,\ \ L_1 = 110$

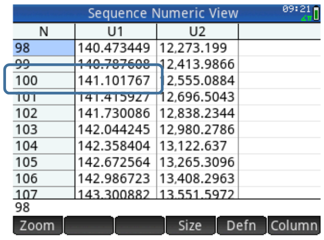


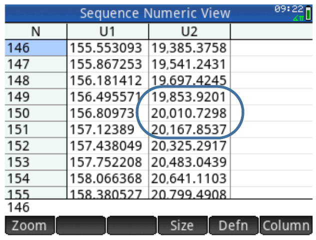


2. Each winding increases the radius by the thickness of the tape (the diameter by twice the thickness).
increase = C of outer layer − C of previous layer

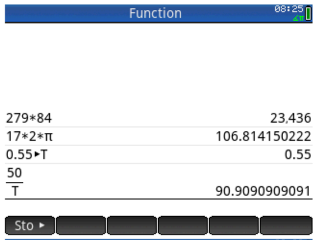
$$= 2\pi(r+t) - 2\pi r$$
$$= 2\pi t$$
$$= 0.1\pi$$

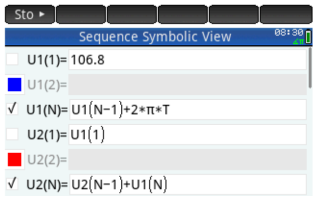
3. a) 141.1mm
b) 12.5 m
c) 150 windings
d) 40 m





4. a) 23.4 m
b) The roll is n layers thick, nt is the thickness of n layers. This equals the difference between the radius of the complete roll and the radius of the cardboard roll, i.e. $R - r$ which is 50 mm.
c) Each layer is approximately 0.55mm thick
The length is also the sum of the arithmetic sequence with n layers. The first layer is $2\pi \times 17$ mm long and the outer layer is $2\pi \times 67$ mm long. So





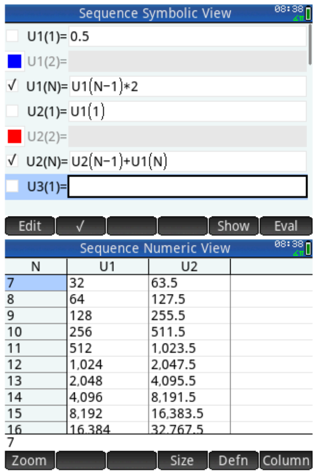
d) (i)

$$L = \frac{n}{2}\left(2\pi \times 17 + 2\pi \times 67\right)$$
$$23436 = \pi n\left(17 + 67\right)$$
$$23436 = 84\pi n$$

ii) $n = 89,\ \ t = 0.56$ mm

Activity 17 Paper and rice

1. a)



Cut	Base	Height of stack
0	10 m × 10 m	0.5 mm
1	10 m × 5 m	1 mm
2	5 m × 5 m	2 mm
3	5 m × 2.5 m	4 mm
4	2.5 m × 2.5 m	8 mm
5	2.5 m × 125 cm	1.6 cm
6	125 cm × 125 cm	3.2 cm

b) $a_{n+1} = 2a_n, a_1 = 1$

c) $h = 2^{n-1}$

d) After 12 cuts

2.

a)

Square	Grains of rice G_n	Total number of grains T_n
1	1	1
2	2	3
3	4	7
4	8	15
5	16	31
6	32	63

b) $G_{n+1} = 2G_n, G_1 = 1$

c)

i) 14th square

ii) 20 kg / 25 mg = 800 000 grains. 20th square.

d) $G_n = 2^n$

e)

i) 31st square: 2.1×10^9 grains = 310 000 cups = 77 000 L = 77 m^3
A silo radius 2 m, height 6 m

ii) 45th square: 3.5×10^{13} grains = 1.3×10^6 m^3. An area of a 10 hectares filled to a depth of 13 m.

f)

i) $T_{n+1} = T_n + 2^n, T_1 = 1$

ii) $T_n = 2^n - 1$

Activity 18 Average speed

1.

a)

Time	Distance (km)
5:00	0
5:15	10
5:30	20
5:45	20
6:00	40

b)

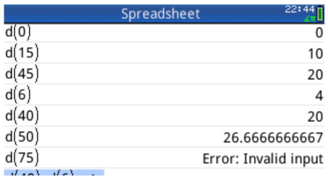
i) 40 km/h

ii) 80 km/h

iii) 40 km/h

c) 40 km/h

2.



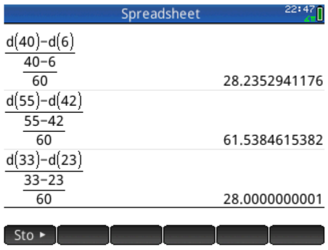
a)

i) 4

ii) 20

iii) 26.7

iv) Undefined as it is outside the domain



b)

i) 28.2

ii) 61.5

iii) 28

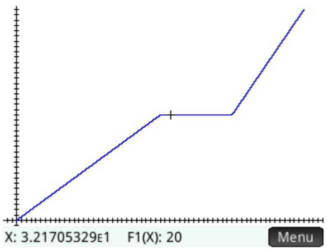
c)

$$\text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

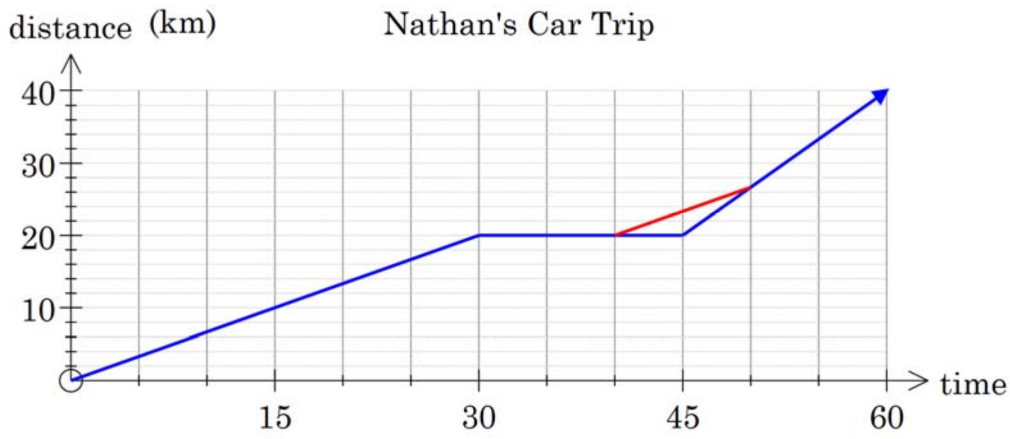
$$= \frac{\text{end distance - start distance}}{\text{(end time - start time) (in hours)}}$$

$$= \frac{d(\text{finish}) - d(\text{start})}{\left(t_{\text{finish}} - t_{\text{start}}\right)/60}$$

d)



e)



f)

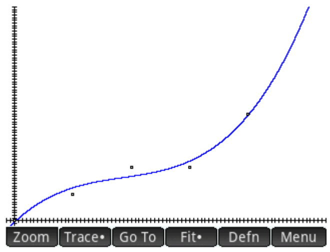
i) $\frac{2}{3}$

ii) $y = \frac{2(x-10)}{3}$

g) He may have had to stop for traffic, take time to accelerate to cruising speed etc

3.

a) $0.000494x^3 - 0.0413x^2 + 1.365x - 0.571$



b)

i) 1.7

ii) 20

iii) 22

iv) 150

c)

i) 32

ii) 35

iii) 21

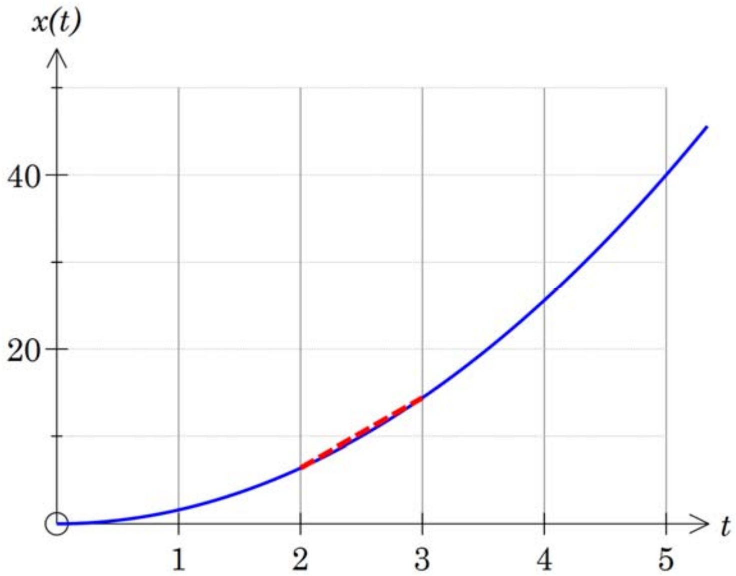
Activity 19 Speed at an instant

1.

Scenario	Graph	Equation
A	2	(iii)
B	3	(ii)
C	4	(i)
D	1	(iv)

2.

a)



b)

Interval	Position (start)	Position (end)	Distance travelled	Average speed
0 – 1	0	1.6	1.6	1.6
0 – 3	0	14.4	14.4	4.8
2 – 3	6.4	14.4	8	8
2.5 – 3	10	14.4	4.4	8.8
2.9 – 3	13.456	14.4	0.944	9.44
3 – 3.1	14.4	15.376	0.976	9.76

c) Dotted line on graph in a)

d) 9.6 m/s

3.

a)

Run	Rise	Slope
1	11.2	11.2
0.5	5.2	10.4
0.1	0.976	9.76
0.05	0.484	9.68
0.01	0.09616	9.616
0.0001	9.60016×10^{-4}	9.60016

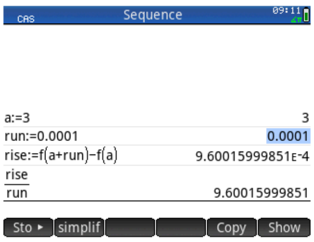
b) The slope is getting closer to 9.6

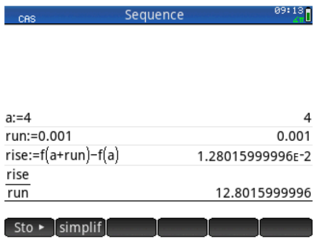
c)

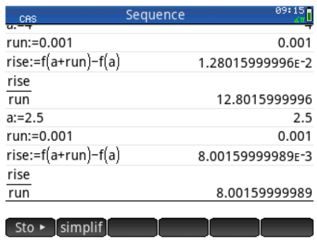
i) 9.6 m/s

ii) 12.8 m/s

iii) 8 m/s







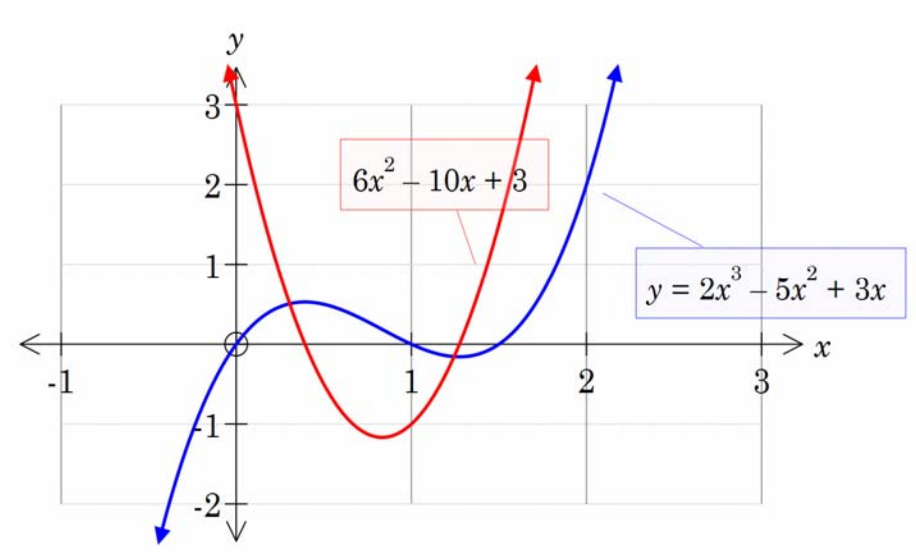
Activity 20 Gradient functions

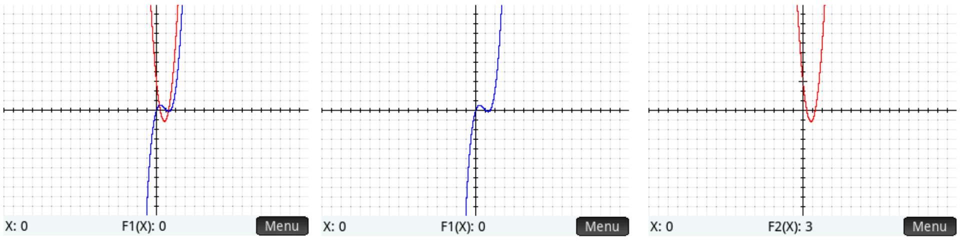
1.

a) (0,0), (1,0), (1.5,0)

b) Local max at (0.392, 0.528) and local min at (1.274, –0.158).

c)





2.

a) (0.392, 0) and (1.275, 0)

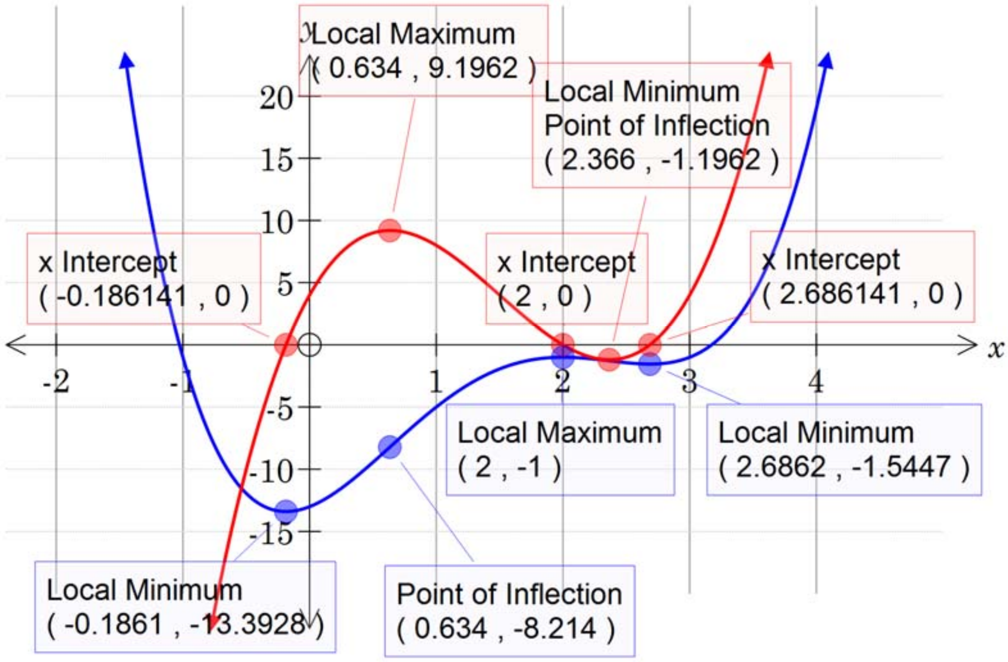
b) Local min at (0.8333, –1.167)

c) Shown on graph in Q1

3. The x-coordinates of the turning points of $y = f(x)$ are the same as the x-intercepts of the slope function. This is because the slope at the turning points is 0.
The local min of the slope function is the minimum gradient, the steepest backward slope. This is a point of inflection of $y = f(x)$

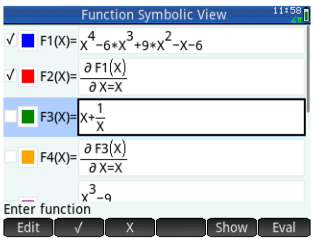
4.

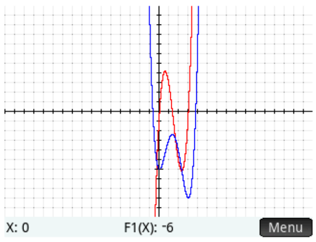
a)



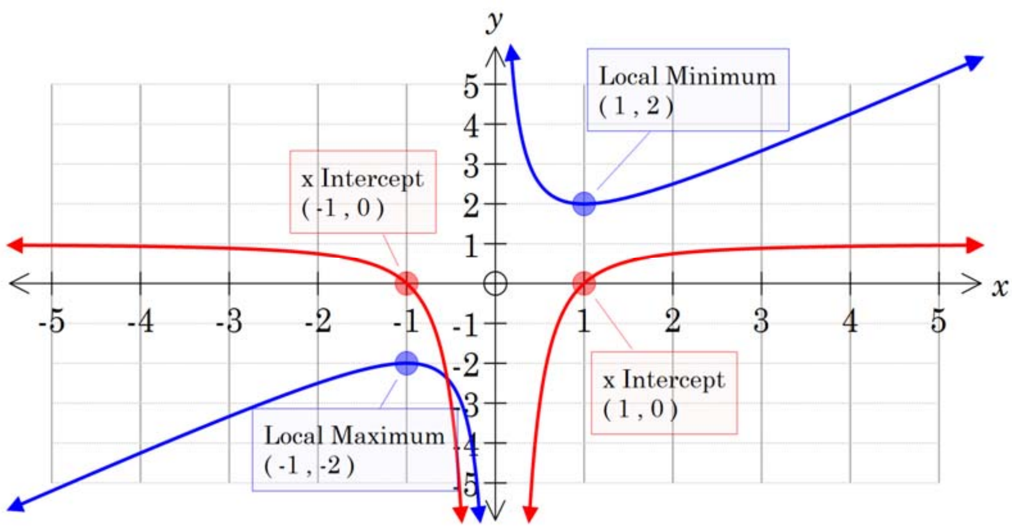
As $x \to \infty,\ f(x) \to \infty,\ f'(x) \to \infty$

$x \to -\infty,\ f(x) \to \infty,\ f'(x) \to -\infty$



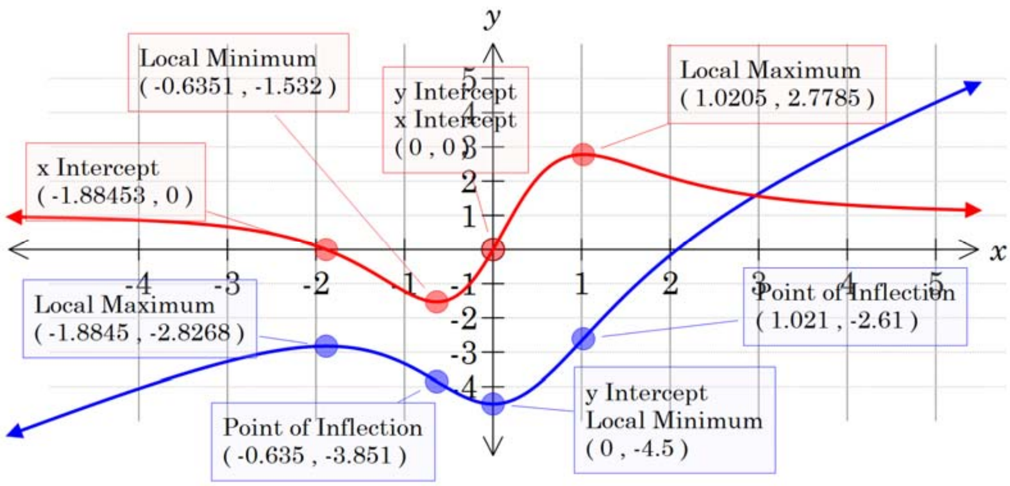


b)

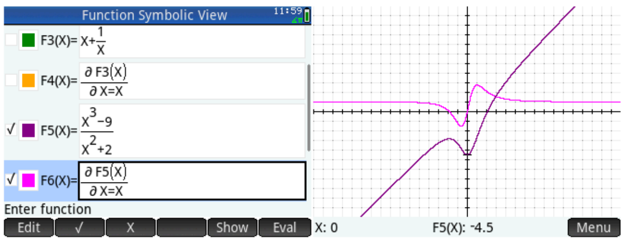


As $x \to \infty,\ f(x) \to \infty,\ f'(x) \to 1$
$x \to -\infty,\ f(x) \to -\infty,\ f'(x) \to 1$

c)



As $x \to \infty,\ f(x) \to \infty,\ f'(x) \to 1$
$x \to -\infty,\ f(x) \to -\infty,\ f'(x) \to 1$

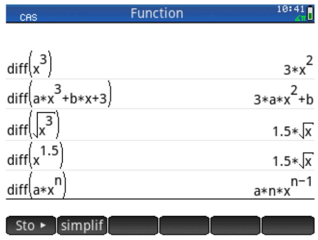


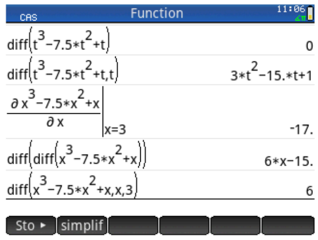
5.

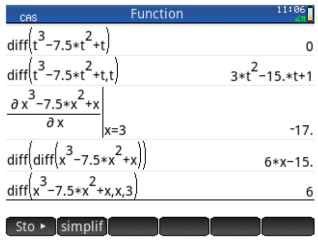
Feature of function	Corresponding feature(s) of derivative function
x-intercept	none
Local maximum	x-intercept, slope goes from positive to negative
Local minimum	x-intercept slope goes from negative to positive
Turning point	x-intercept slope is 0
Point of inflection	Turning point

1.

$\dfrac{d}{dx}x^3$	$3x^2$
$\dfrac{d}{dx}ax^3+bx+3$	$3ax^2+b$
$\dfrac{d}{dx}\sqrt{x^3}$	$\dfrac{3x^2}{2\sqrt{x^3}}$
$\dfrac{d}{dx}x^{1.5}$	$\dfrac{3}{2}\sqrt{x}$
$\dfrac{d}{dx}(ax^n)$	anx^{n-1}
$\dfrac{d}{dx}x^3-7.5x^2+x$	$3x^2-15x+1$
$\dfrac{d}{dx}t^3-7.5t^2+t$	0
$\dfrac{d}{dt}t^3-7.5t^2+t$	$3t^2-15t+1$
$\dfrac{d}{dx}x^3-7.5x^2+x\Big\|_{x=3}$	-17
$\dfrac{d^2}{dx^2}x^3-7.5x^2+x$	$6x-15$
$\dfrac{d^2}{dx^2}x^3-7.5x^2+x$	$6x-15$
$\dfrac{d^3}{dx^3}x^3-7.5x^2+x$	6







2.

a) $8x-5$

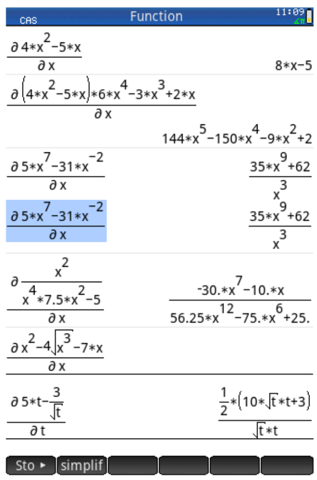
b) $144x^5-210x^4+60x^3+24x^2-20x$

c) $35x^6+\dfrac{62}{x^3}$

d) $\dfrac{-\left(8x^5+40x\right)}{\left(2x^4+15x^2-10\right)^2}$

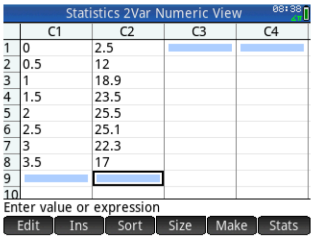
e) $2x-\dfrac{3}{4}x^{\frac{-1}{4}}-7$

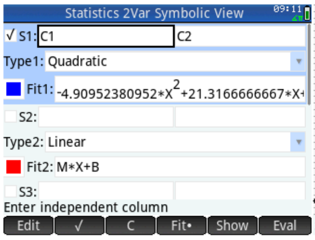
f) $\dfrac{3}{2t^{1.5}}+5$

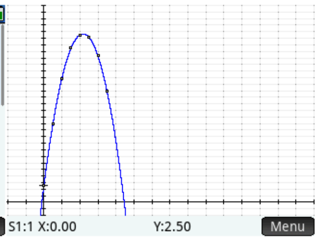


Activity 22 Modelling motion

1. $h(t) = -4.9t^2 + 21.3t + 2.53$

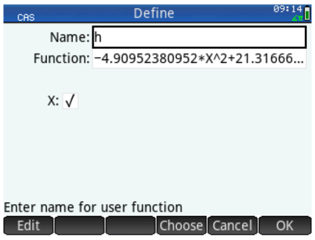






2.

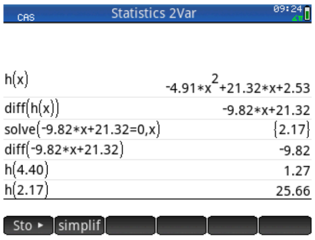
$$v(t) = \frac{dh}{dt}$$
$$= -9.8t + 21.3$$



3. 2.17 s

4.

$$a(t) = \frac{dv}{dt} = -9.8$$



5. 21.3

6. 21.9 m/s after 4.4 s

7. 25.7 m